The problem asks for the induced thermal stress of a solid steel shafting subjected to a temperature rise from **35º** to **120º**.

If a body is free to expand or contract, no stress is induced. If expansion or contraction is restricted, thermal stresses develop."

The problem statement fails to explicitly state that the shaft is completely restricted or fixed between two rigid walls. If the shafting were simply lying freely on a shop floor, the induced thermal stress would be **0 MPa**. Furthermore, standard nominal properties for structural carbon steel must be pulled from the provided reference material constraints: Modulus of Elasticity (E) is taken as 200GPa (200 x 10^3 MPa) and the coefficient of linear thermal expansion (α) for steel is exactly 10.8 x 10^{-6} m/m-ºC.

<u>Step 1:</u> Geometric Expansion and Thermal Strain Formulation

When a material experiences a temperature shift, its internal kinetic energy increases, forcing the atomic lattice structure to expand. The unconstrained thermal deformation (δ_{th}) is directly proportional to its initial length (L_o), the temperature differential (ΔT), and its material-specific expansion rate (α).

Mathematically, this unhindered elongation is expressed as:

$$\delta_{th} = \alpha L_o\left(t_2 - t_1\right)$$

However, when rigid limits prevent this physical movement, the walls exert an equal and opposite mechanical reaction force (F) against the expanding shaft. This force creates a compressive mechanical deformation (δ_{mech}) that completely neutralizes the thermal expansion. According to Hooke's Law, this axial mechanical deformation is calculated as:

$$\delta_{mech} = \frac{FL_o}{AE}$$

Because the total net physical change in length equals zero $\left(\delta_{net} = \delta_{th} - \delta_{mech} = 0\right)$, we set the two expressions equal to each other:

$$\alpha L_o\left(t_2 - t_1\right) = \frac{FL_o}{AE}$$

<u>Step 2:</u> Isolating for Induced Stress (S)

Notice that the initial length (L_o) appears on both sides of our equilibrium equation. Dividing both sides by L_o eliminates the length parameter from the equation entirely. This is a vital conceptual realization for board exam questions: **the induced thermal stress in a completely restricted, uniform member is entirely independent of its length and cross-sectional area.**

Recall that normal stress (S) is defined as force per unit area $\left(S = \frac{F}{A}\right)$ Substituting S into the remaining terms yields the classic thermal stress equation:

$$\alpha\left(t_2 - t_1\right) = \frac{S}{E}$$

Isolating the induced stress (S), we get:

$$S = \alpha E\left(t_2 - t_1\right)$$

<u>Step 3:</u> Numerical Substitution and Calculation

We can now plug our values into the isolated thermal stress formula.

- α = 10.8 x 10^{-6} /^{o}C
- E = 200 GPa = 200,000 MPa
- t_1 = 35^{o}C
- t_2 = 1

$$S = \left(10.8\,x\,10\,-\,6\,/\ {}^{o}C\right)(200,000\,MPa)(120\,-\,35)\ {}^{o}C$$

$$S = 183.6\,MPa$$

The most profound lesson to take away from this problem is the invisible, immense power of thermal energy. By simply heating this stationary, fixed piece of steel by 85oC, a massive internal compressive stress of **183.6 MPa** is generated. For a 6 cm diameter shaft, this translates into a crushing physical force of over **524 kN** (roughly 118,000 lbs of force) pushing outward against the boundaries.