

This question presents a classic unit mismatch intentionally designed to test a board examinee's conversion proficiency: the shaft diameter is specified in **inches** (1.5 inches) and the torque varies in **inch-pounds** (in - lb), whereas all the multiple-choice options are listed in **kiloPascals** (kPa).

The problem asks for the **variable component stress** (also known as the alternating or amplitude shear stress, S_a). This parameter isolates the fluctuating wave amplitude from the steady baseline mean stress. To obtain an exact match with the metric board choices, the most precise path is to compute the extreme stress states (S_{max} and S_{min}) directly in English units (psi), find their fluctuating components, and then convert the final stress threshold to kilopascals (kPa)

Step 1: Calculation of Extreme Stress States (S_{max} and S_{min})

According to solid mechanics and elastic torsion theory, the shear stress at the outermost fiber of a solid cylindrical shaft is directly proportional to the applied torque (T) and the outer radius ($c = \frac{D}{2}$), and inversely proportional to the polar moment of inertia ($J = \frac{\pi D^4}{32}$). Combining these geometric constraints yields the standard solid shaft torsion equation:

$$S = \frac{16T}{\pi D^3}$$

We will now use this relationship to evaluate the maximum shear stress (S_{max}) corresponding to $T_{max} = 8600$ in-lb, and the minimum shear stress (S_{min}) corresponding to $T_{min} = 2800$ in-lb, using the given diameter $D = 1.5$ in:

- **Maximum Shear Stress (S_{max}):**

$$S_{max} = \frac{16T_{max}}{\pi D^3} = \frac{16(8,600 \text{ in-lb})}{\pi(1.5 \text{ in})^3} = 12,977.61 \text{ psi}$$

- **Minimum Shear Stress (S_{min}):**

$$S_{min} = \frac{16T_{min}}{\pi D^3} = \frac{16(2,800 \text{ in-lb})}{\pi(1.5 \text{ in})^3} = 4,225.27 \text{ psi}$$

Step 2: Isolation of the Variable Component Stress (S_v)

A cyclic or fluctuating stress waveform can be broken down into two distinct mathematical components: a steady-state average value (mean stress, S_m) and an alternating peak amplitude (variable stress, S_v). The variable component represents the pure dynamic amplitude of the fluctuation.

Mathematically, the variable stress component is defined as half of the total stress range between the maximum and minimum limits:

$$S_v = \frac{S_{max} - S_{min}}{2} = \frac{(12,977.61 - 4,225.27) \text{ psi}}{2} = 4,376.17 \text{ psi}$$

Step 3: Unit Transformation to Kilopascals (kPa)

To align with the metric multiple-choice options provided on the exam, we must convert our final variable component stress from pounds per square inch (psi) to kiloPascals (kPa). Using the exact standard conversion factor where 1 psi = 6.894757 kPa:

$$S_v = 30,172.63 \text{ kPa}$$

This variable stress component (S_v) is the primary driving input variable used when evaluating dynamic machine longevity via standard fatigue equations such as the Soderberg, Goodman, or Gerber fatigue criteria. To extend the operating life of a fluctuating pump or drivetrain shaft in an industrial plant, engineers will actively try to reduce the variable stress component by adding a flywheel to smooth out torque spikes or by widening the shaft diameter.