When sizing a mechanical component, it must satisfy two independent structural criteria:

1. **Strength Criterion:** The actual induced stress (S_T) must not exceed the allowable design stress ($S_{all} = 130\ MPa$).
2. **Rigidity Criterion:** The total elastic deformation (δ) must not exceed the maximum operational clearance limit ($\delta_{all} = 6mm$).

To solve this completely, we must calculate the required diameter under both independent constraints separately. The larger of the two computed diameters will serve as the final safe design size.

<u>Part 1:</u> Sizing Based on the Strength Criterion (Stress Limit)

Axial tensile stress is defined as the internal resistance force distributed uniformly per unit of cross-sectional area. According to basic stress analysis principles

$$S_T = \frac{F}{A} = \frac{F}{\frac{\pi}{4}D^2}$$

$$130\frac{N}{mm^2} = \frac{3{,}000\ N}{\frac{\pi}{4}D^2}$$

$$D = 5.42\ mm$$

This means the wire must be at least 5.42 mm thick to prevent structural yielding or breaking under the load.

<u>Part 2:</u> Sizing Based on the Rigidity Criterion (Elongation Limit)

According to Hooke's Law, total linear deformation (δ) within the material's elastic limit is directly proportional to the force (P) and length (L), and inversely proportional to the area (A) and stiffness modulus (E):

$$\delta = \frac{FL}{AE} = \frac{FL}{\left(\frac{\pi}{4}D^2\right)E}$$

$$6\ mm = \frac{(3{,}000\ N)(12{,}000\ mm)}{\left(\frac{\pi}{4}D^2\right)\left(220{,}000\ \frac{N}{mm^2}\right)}$$

D = 5.89 mm

This means the wire must be at least 5.89 mm thick to prevent stretching more than the 6 mm baseline.

If we select 5.42 mm, the wire will be strong enough but it will stretch beyond 6 mm}, violating the rigidity limit. Therefore, the larger diameter governs the design. Rounding up to the nearest standard nominal millimeter choice provided in the options:

D = 6 mm

This problem highlights one of the most fundamental principles of industrial machine design: **a component can be perfectly strong enough to carry a load, yet fail completely due to excessive flexibility.**