MACHINE AND STRUCTURE

In the study of mechanical design, it is essential to distinguish between a **structure** and a **machine**. A **structure** is a combination of resistant bodies intended to support loads or carry weights while remaining perfectly static. Its defining characteristic is that there is **no relative motion** between its individual parts; its primary purpose is to resist stress and maintain stability, such as in a bridge or a building frame. Conversely, a **machine** is an assembly of resistant bodies where the parts are designed to **move relative to each other**. The goal of a machine is to transmit power and perform work by transforming one form of energy into another, such as the way an engine converts fuel into rotation.

The fundamental difference between these two entities is best explained through the equation for **Mechanical Work**:

$$W = F \cdot d \cos\theta$$

F represents the applied force, **d** is the displacement, and **cosθ** accounts for the angle between the force and the direction of motion. For a **structure**, even if the force applied is massive (like the weight of a skyscraper on its foundation), the displacement (d) is zero. Because you are multiplying by zero, the resulting work (W) is also zero. A **machine**, however, is specifically designed to create displacement. By moving a component over a distance, a machine successfully transfers energy and performs measurable mechanical work.

The term **cosθ** in the work equation dictates that only the portion of the force that acts in the same direction as the motion counts as work. This highlights a key engineering principle: for a machine to be efficient, the force must be aligned with the intended displacement. While structures are designed to **hold** force, machines are designed to **move** force, using displacement to turn input energy into useful output.

If a structure "moves," it has failed its design purpose. If a machine "stops moving," it has failed its design purpose. Everything in between is the management of **Mechanical Work**.

When a force acts at 180° to the direction of displacement, $\cos(180^\circ) = -1$, resulting in **negative work** because negative work removes kinetic energy from the system and converts it into heat. This explains why we use lubricants or ball bearings: we aim to minimize the negative work of friction so that more of our input force can be converted into useful output work.

In Machine Design, our primary objective is to facilitate the transfer of energy by manipulating the relationship between **Force (F)** and **Displacement (d)**. We achieve this through the principle of **Mechanical Advantage**, which allows us to trade displacement for force—enabling a machine to move a heavy load using a smaller input effort. By analyzing these components, we can maximize a machine’s output and efficiency, ensuring that the energy provided is effectively converted into useful motion rather than being lost to heat or friction.

As we apply forces to transmit power, the components of our machine experience internal resistance known as **stress**. This is where the **Selection of Materials** becomes critical. We must choose materials with mechanical properties—such as yield strength, ductility, and fatigue resistance—that can withstand these stresses without undergoing permanent deformation or failure. Therefore, Machine Design is a balancing act: we must design for enough displacement to perform work, while ensuring the material is robust enough to handle the forces required for that movement.

STRESS ANALYSIS

Machine Design requires knowledge of how materials behave under different forces. Stress analysis provides insight into tensile, compressive, shear, and torsional stresses, all of which influence material selection, component sizing, and safety considerations in machine design.

STRENGTH OF MATERIALS

Strength of materials is a measurement of the material's ability to resist or withstand forces that result from stress and strain.

Stress

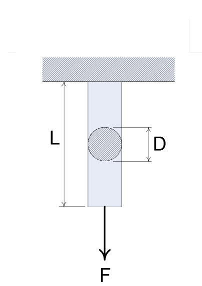
Stress represents the **internal resistance** of a material to deformation under an external load. Changes in stress occur because the material adjusts to varying force and area conditions. If stress exceeds the material's strength, deformation or failure (like cracking or breaking) can occur.

Relationship Between Stress and Strength:

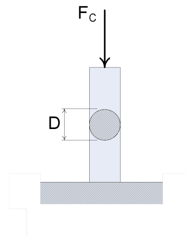
- Stress tells you how much force per unit area is acting on a material at a given moment.
- Strength defines the limit of stress the material can withstand without failure.

Stress Equations:

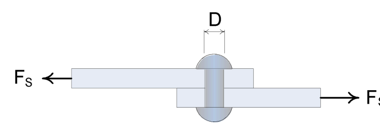
1. **Tensile stress:** Occurs when a material is subjected to a force that pulls it apart.

$S_t = \frac{F}{A}$	

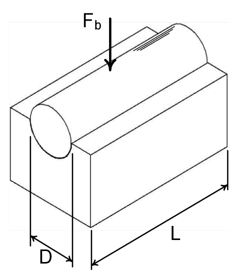
2. **Compressive stress:** Results from a force that compresses or squeezes the material.

$S_C = \frac{F}{A}$	

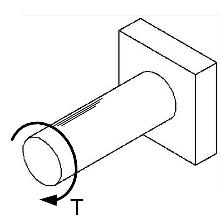
3. **Shearing stress:** Occurs when a force causes layers of the material to slide against each other.

$S_S = \frac{F}{A}$	

4. **Bearing stress:** The contact pressure between two surfaces. Bearing stress, also referred to as **crushing stress**, is a localized compressive stress that occurs at the surface of contact between two machine parts that are in relative motion or are stationary under a load. It typically appears in areas where one component presses against another, such as in riveted joints, cotter joints, or journal bearings.

$S_B = \frac{F}{A} = \frac{F}{LD}$	

5. **Torsional stress:** Associated with twisting, it occurs when torque is applied around the shaft axis.

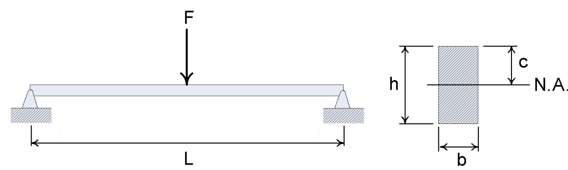
$S_S = \frac{Tc}{J}$	

T = torque
c = distance of the outermost fiber from the neutral axis.
Equal to the radius, r or D/2 for circular shaft
J = polar moment of inertia

- Table AT1 Properties of Sections: from Page 563 Design of Machine Elements 4th Edition by Virgil Faires

For circular Solid Shaft:	For circular Hollow Shaft:
$J = \frac{\pi}{32} D^4$	$J = \frac{\pi}{32}\left(D_o^{\ 4} - D_i^{\ 4}\right)$
$S_S = \frac{16T}{\pi D^3}$	$S_S = \frac{16T}{\pi\left(D_o^{\ 4} - D_i^{\ 4}\right)} D_o$

6. **Flexural or Bending stress:** Occurs when a material is subjected to bending forces, often relevant in beams and other structural components.

$S_f = \frac{Mc}{I}$	

M = moment or bending moment
c = distance of the outermost fiber from the neutral axis
I = moment of inertia about the neutral axis

For Rectangular Cross Section: $I = \frac{bh^3}{12}$

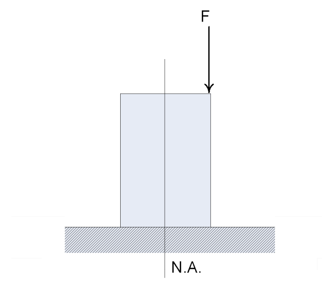
- Table AT1 Properties of Sections: from Page 563 Design of Machine Elements 4th Edition by Virgil Faires

Section Modulus: $z = \frac{I}{c}$

The **section modulus** is a measure of the strength of a beam's cross-section in resisting bending. It relates the **moment of inertia** (I) and the **distance from the neutral axis** to the outermost fiber (c). A **larger section modulus means less bending stress** for the same applied moment.

For Rectangular Cross Section: $z = \frac{bh^2}{6}$

7. **Combined stress:** Situations where different types of stress act simultaneously on a material, necessitating combined stress analysis.

$S = \frac{F}{A} \pm \frac{Mc}{I}$	

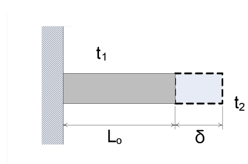
± indicates that one side of the cross-section experiences **tensile (+)** or **compressive (-)** stress depending on the location in the cross-section.

8. **Thermal Stress**

Thermal stresses are internal forces that develop in a material when there is a change in temperature, but the material's expansion or contraction is restricted. These stresses arise because most materials expand when heated and contract when cooled. If this movement is prevented, stress is induced.

Key Concept:

- If a body is free to expand or contract, no stress is induced.
- If expansion or contraction is restricted, **thermal stresses** develop.
- The stress depends on the material's modulus of elasticity, coefficient of thermal expansion, and the magnitude of temperature change.



Linear expansion: $\delta = \alpha L_O(t_2 - t_1)$

Where:

δ = thermal elongation

α = coefficient of linear thermal expansion, m/m-°C

For Steel: $10.8 \times 10^{-6} \frac{m}{m - {}^oC}$

L_o = original length

t_2 = final temperature, °C

t_1 = initial temperature, °C

Thermal stress: $\delta = \frac{F}{A}\frac{L}{E}$

$$\alpha L_O(t_2 - t_1) = \frac{F}{A}\frac{L_O}{E}$$

$$S_T = \alpha E(t_2 - t_1)$$

HOOKE'S LAW

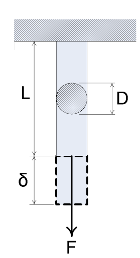
Selecting the right material for engineering components is essential, as it affects durability, performance, and cost. Engineers must consider a material's mechanical properties, geometry, and the intended manufacturing process. A critical part of material selection is conducting **tension and compression tests**, which help identify the strength, yield point, and elasticity of materials, all key factors for determining suitability for different applications.

Stress

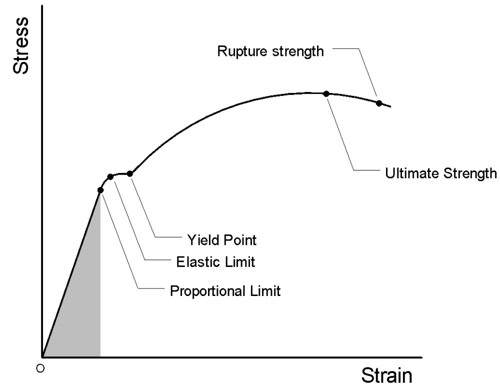
This is the internal force per unit area within a material that resists deformation under load. Types of stress include **tensile**, **compressive**, **shear**, **bearing**, and **torsional** stresses. Each has distinct effects on material performance and requires specific calculation methods.

Strain, ε

Defined as deformation per unit length under load, strain indicates how much a material stretches or compresses, essential for evaluating material elasticity and ductility

$\varepsilon = \frac{\delta}{L}$	

The stress-strain curve:



Hooke's Law:

Hooke's Law states that within proportional limit the stress in the material is directly proportional to the strain. This law was observed by Robert Hooke in 1678.

Restrictions:

- The load must be axial.
- The bar must have a constant cross sectional area and must be homogenous.
- The stress must not exceed the proportional limit.

$$S \, \alpha \, \varepsilon$$

$$S = E\varepsilon$$

$$\frac{F}{A} = E\frac{\delta}{L}$$

$$\delta = \frac{F}{A}\frac{L}{E}$$

E = Young's modulus or modulus of elasticity
For steel: E = 30 x 10^6 psi = 207 GPa

Hooke's Law generally applies to any elastic material that deforms linearly under an applied force. This includes materials like rubber bands, springs, beams, or even metals, as long as they are within their "elastic limit" — the range in which they return to their original shape after the force is removed.

Terminologies

Modulus of Elasticity (E)

A measure of material stiffness, used to predict deformation under load. High values indicate stiffer materials, essential for load-bearing applications.

Elastic limit

Elastic limit is a limit beyond which the material will no longer go back to its original shape or form upon the removal of the load.

Yield point

Yield point is the point where the material experiences an appreciable elongation or yielding without any increase in load.

Ultimate strength

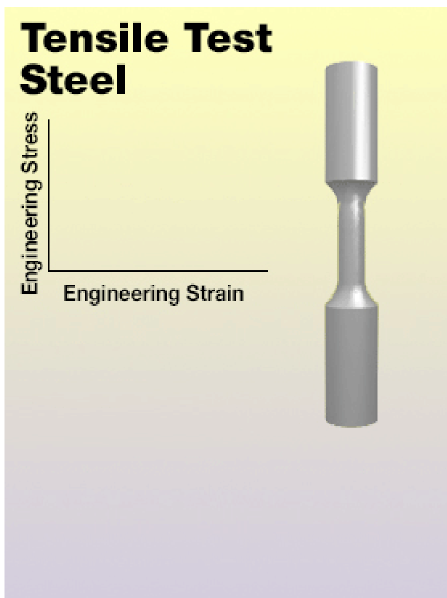
Ultimate strength is the maximum point in the stress-strain diagram the material can reach without rupture or breaking.

Rupture strength

Rupture strength or also known as breaking strength is the strength of material at failure.

Necking

Necking or narrowing is the reduction in the cross sectional area of a steel specimen at failure.



Materials can be classified as:

Ductile Materials: The ability of a material to be stretched into a wire under tensile stress. (e.g., mild steel).

Brittle Materials: Opposite of ductility; the tendency to fracture without significant deformation. (e.g., cast iron).

The **mechanical properties of steels** vary by carbon content and alloying elements, impacting their tensile strength, yield strength, ductility, and hardness. Common types include:

- **Low Carbon Steel**: Good ductility and machinability.
- **Medium Carbon Steel**: Higher strength than low carbon but less ductile.
- **High Carbon Steel**: High strength and hardness but more brittle.

Alloy steels are modified with additional elements to enhance specific properties. Some key alloying elements include:

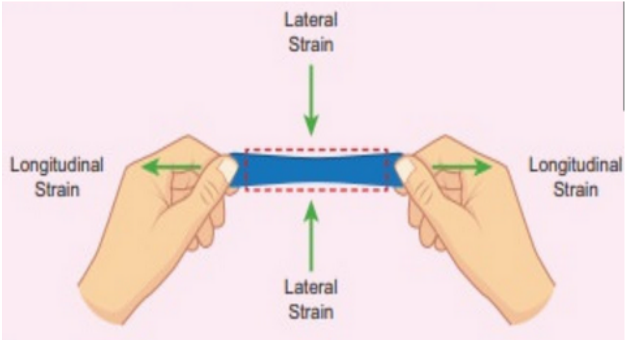
- **Chromium (Cr)**: Forms chromium carbide, which increases hardness and wear resistance. Often used in amounts between 0.5% and 1.5% for applications needing durability.
- **Nickel (Ni)**: Enhances strength and toughness without sacrificing ductility. Nickel is commonly used in amounts of 2% to 5%, and it lowers the carbon content in the eutectoid steel, increasing temperature resistance.
- **Molybdenum (Mo)**: Adds hardness and toughness, making steel suitable for high-stress environments. Used in small quantities (0.15% to 0.30%) for applications where durability is crucial.
- **Vanadium (V)**: Improves grain structure and strength. Typically used in tool steels due to its strength and deoxidizing properties, though usually in small amounts (less than 0.2%).
- **Tungsten (W)**: Known for retaining hardness at high temperatures, tungsten is ideal for high-strength applications like cutting tools.
- **Manganese (Mn)**: Enhances strength and hardness by dissolving in ferrite and forming carbides. Manganese content above 1% classifies it as a manganese alloy, adding resilience in dynamic applications.
- **Silicon (Si)**: Acts as a deoxidizer and increases strength, especially in magnetic and electrical applications.

Poisson's Ratio

Poisson's Ratio is a material property that represents the ratio of **lateral strain** to **linear strain** when a material is subjected to stress within its elastic limit. It is denoted by μ. The ratio is named after the French mathematician and physicist Siméon Poisson.

Mathematical Representation: $\mu = \frac{Lateral\ Strain}{Linear\ Strain}$

When a material is stretched in one direction, it contracts in the perpendicular direction, and vice versa. The extent of this contraction or expansion relative to the applied force is quantified by Poisson's Ratio.



Where:

- **Lateral Strain**: The deformation (strain) in the direction perpendicular to the applied force.
- **Linear Strain**: The deformation (strain) in the direction of the applied force.

Values of Poisson's Ratio for Commonly Used Materials

Material	Poisson's ratio, μ
Carbon Steel	0.292
Cast iron (gray)	0.211
Copper	0.326
Brass	0.324
Aluminum (all alloys)	0.333
Concrete	0.08 to 0.18
Rubber	0.45 to 0.50

FACTOR OF SAFETY

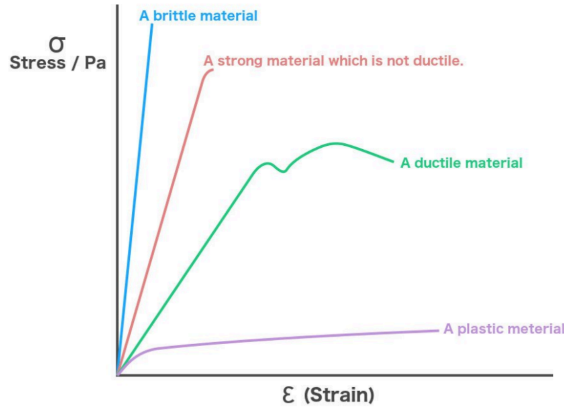
Factor of safety is used to provide an allowance over the theoretical material capacity in order to accommodate uncertainties in the design process. It is defined as the ratio of the stress that will cause failure, S_{fail} to the allowable stress, S_{all}.

$$FS = \frac{S_{fail}}{S_{all}}$$

Working stress, also known as **safe stress** or **allowable stress**, is the stress level at which a material can safely operate under normal working conditions without failure. It is always less than the material's **ultimate stress** or **yield stress** to provide a margin of safety.

Allowable Stress, S_{all}

Allowable stress is the level in which the stress values will be limited to or not exceed the proportional limit. Allowable stress can either be taken as yield stress or ultimate strength divided by the factor of safety.



Design stress based on yield strength, S_y: Ensures the material does not permanently deform

$$S_{all} = \frac{S_y}{FS}$$

For ductile materials, the design stress is calculated based on the Yield Strength, such as structural steel (AISI 1020), transmission shafting (AISI 1045), aluminum alloys, and copper, have the ability to stretch or deform plastically before they actually break. In precision machine design, however, permanent deformation is considered a failure. If a steel shaft bends or a bolt stretches permanently, the machine loses alignment and functionality even if the part hasn't snapped. Therefore, we set the safety limit at the yield point to ensure the material remains in its elastic region and returns to its original shape after the load is removed.

Design stress based on ultimate strength, S_u : Ensures the material does not completely fail.

$$S_{all} = \frac{S_u}{FS}$$

Brittle materials are designed based on the Ultimate Strength (Su). Materials in this category, such as Gray Cast Iron (commonly used for engine blocks and machine beds), concrete, glass, and ceramics, do not exhibit a distinct yield point or plastic deformation phase. Instead of stretching, they resist the load rigidly until they reach their breaking point, at which they fail suddenly and catastrophically (fracture). Since there is no "yielding" to serve as a warning or a functional limit, the design must be based on the ultimate breaking strength, usually with a significantly higher Factor of Safety to account for their unpredictable nature and lack of toughness.

The **Factor of Safety** ensures a margin of error in design to account for uncertainties, material inconsistencies, and unforeseen operating conditions. A higher FS means greater reliability but also increased material usage and cost. Engineers must state **what strength basis** they use and consider non-linear stress behaviors in certain applications.

Designing based yield strength is generally preferred in design because it prevents permanent deformation. If a component operates beyond its yield strength (S_y), it will not return to its original shape, leading to functional failure. Designing based on yield strength also provides a safety margin, ensuring that even under high loads, the material remains elastic and predictable, reducing the risk of unexpected failure.

There are some cases where ultimate strength (S_u) can be used in design. Brittle materials, such as glass, ceramics, and cast iron, do not have a distinct yield point, so they are designed based on ultimate strength instead of yield strength. Additionally, one-time use components, like automobile crumple zones and certain safety devices, are intentionally designed to deform permanently to absorb energy during impact.

Recommended Values of Factor of Safety

- Table 1.1 FACTORS OF SAFETY (DESIGN FACTORS): from Page 20 Design of Machine Elements 4th Edition by Virgil Faires

TABLE 1.1 FACTORS OF SAFETY (DESIGN FACTORS)

The factors of safety marked with * are primarily for beginners' use, although they are traditional values. They should not be used when a detailed accounting is made of the variable loading, stress concentrations, etc., Chapter 4. Acceptable for use with typical strengths.

KIND OF LOAD	STEEL, DUCTILE METALS		CAST IRON, BRITTLE METALS	TIMBER
	Based on Ultimate Strength	*Based on Yield Strength*	*Based on Ultimate Strength*	
Dead load, $N =$	3–4	1.5–2	5–6	7
Repeated, one direction, gradual (mild shock),* $N =$	6	3	7–8	10
Repeated, reversed, gradual (mild shock),* $N =$	8	4	10–12	15
Shock,* $N =$	10–15	5–7	15–20	20

What is Design Factor, N_d?

The design factor is a predetermined safety margin chosen **before** manufacturing. It accounts for uncertainties in material properties, loading conditions, and environmental effects.

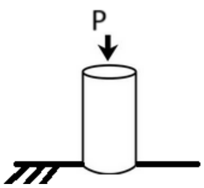
$$N_d = \frac{S_{nominal}}{\sigma_{nominal}}$$

Where: N_d = design factor
$S_{nominal}$ = nominal Strength of material
$\sigma_{nominal}$ = nominal Stress of material
= expected stress under normal conditions

Illustration:

A cylindrical column is subjected to an axial load (P) of 3 kips applied at its top. The material used for the column has an ultimate compressive strength, S_u of 45 ksi. To ensure a safe design, the structure must be designed with a design factor, N_d of 2. Find the diameter of the column.

Available standard diameters, inches: $\left\{\frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \ldots\right\}$



Given: Axial Load (P): 3 kips
Ultimate Compressive Strength (Su): 45 ksi
Design Factor (N_d): 2
Standard Sizes available: 1/8"(0.125),1/4"(0.250),3/8"(0.375),1/2"(0.500)

Find: Final standard diameter (d_{final})
Actual Factor of Safety (FS_{actual})

The design factor is the ratio of the nominal strength to the nominal stress:

$$N_d = \frac{S_{nominal}}{\sigma_{nominal}}$$

$$N_d = \frac{S_{nominal}}{\frac{P}{A}} = \frac{A \cdot S_{nominal}}{P}$$

Rearrange to solve for required area A:

$$A = \frac{N_d \cdot P}{S_{nominal}} = \frac{(2)(3{,}000\ lb_f)}{45{,}000\ \frac{lb_f}{in^2}} = 0.1333\ in^2$$

$$A = \frac{\pi}{4}d^2$$

$$0.1333\ in^2 = \frac{\pi}{4}d^2$$

d = 0.4120 - inch

From available standard sizes, select: d = $\frac{1}{2}$" $= 0.5\ in$

The Factor of Safety (FS) is calculated as:

$$FS = \frac{S_{fail}}{S_{all}} = \frac{S_u}{S_{all}} = \frac{S_u}{\frac{F}{\frac{\pi}{4}d^2}} = \frac{45{,}000\ \frac{lb_f}{in^2}}{\frac{3{,}000\ lb_f}{\frac{\pi}{4}(0.5in)^2}}$$

FS = 3.0

Thus, the actual factor of safety, FS=3.0, which is higher than the design factor, N_d=2, ensures that the design is safe.

The theoretical diameter required was 0.412". By rounding up to 0.5", we increased the cross-sectional area by nearly 47%. This significantly lowers the internal stress. Because the area increased, the actual Factor of Safety rose from the required 2.0 to **2.94**.This effect ensures that even if the load slightly exceeds 3 kips, or if the material has minor manufacturing defects, the column will not fail. In engineering, selecting the "next larger standard size" is a standard practice to accommodate the discrepancy between theoretical calculations and real-world material availability.

Key Takeaways:

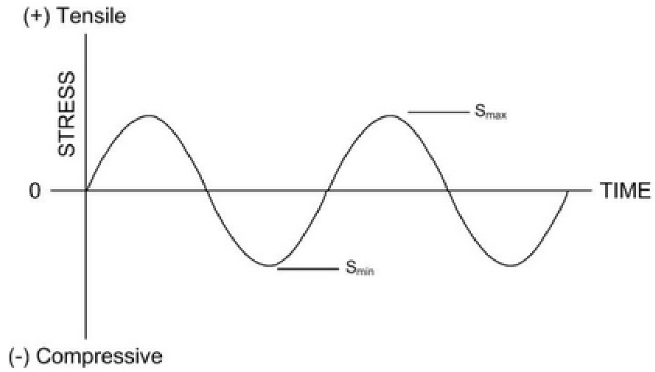
- Design Factor (N_d) is set before construction to determine allowable stress.
- Factor of Safety (FS) is checked after testing to see how much actual safety remains.
- A good design ensures that FS is greater than or equal to the design factor to maintain safety.

Criteria for Selecting FS:

- **Material Properties**:
    - Reliability of material strength under operating conditions.
    - Consistency in material quality.
- **Failure Consequences**:
    - Extent of damage if failure occurs.
    - Loss of life or property in case of failure.
- **Design and Manufacturing**:
    - Accuracy of initial stresses introduced during manufacturing.
    - Simplifying assumptions made in design calculations.
- **Operating Conditions**:
    - Environmental factors such as temperature, corrosion, and wear.
- **Load Characteristics**:
    - Type of loads (static, dynamic, or impact).

TYPES OF LOADS

Understanding the various types of loads that a machine or structure can experience is crucial for ensuring its safety, durability, and performance. Loads can range from static, where forces are constant and unchanging, to fluctuating and shock loads, where forces vary significantly over time. R = stress ratio = S_{min}/S_{max}.



- Static
  Static loads are non-varying loads and applied slowly without creating a shock. Pressure exerted by a rock in a surface or often called as dead load is one example of static load. The resulting stress is called static stress. ($S_{min} = S_{max}$, R = 1.0)

- Repeated and reversed
  A machine member is subjected to a repeated and reversed loading when a certain level of tensile stress is applied followed by the same level of compressive stress and is repeated numerous times. ($S_{min} = -S_{max}$, R = -1.0)

  When loads are applied repeatedly, the **endurance limit** (rather than the yield strength) should be used to determine the factor of safety. Repeated loads can cause fatigue failure, requiring a higher FS for long-lasting reliability.

- Fluctuating or Live
  A machine member is subjected to a fluctuating load or live load when the load varies alternately from maximum value then to zero value and minimum value. The resulting stress is called fluctuating stress.

$$S_m = average\ or\ mean\ stress = \frac{S_{max} + S_{min}}{2}$$

- Shock or Impact
  A machine member is subjected to shock or impact when the loads are applied suddenly and rapidly that cause shock or impact. An example is a weight falling in a concrete structure.

- Random or variable
  A machine member is subjected to random loads when the loads are applied at different amplitudes or in unpredictable manner.

$$S_a = Variable\ or\ alternating\ component\ stress\ = \frac{S_{max} - S_{min}}{2}$$

Stress Ratio, R - used to classify different types of fluctuating loads

$$R = \frac{S_{min}}{S_{max}}$$

R = 1	Static
R = -1	Repeated and reversed
0 < R < 1	Tensile fluctuating load
R = 0	Pulsating load, no compression
R > 1	Purely compressive

In engineering design, understanding the effects of variable loading is critical for ensuring the reliability and longevity of components. Variable loading refers to fluctuations in the loads that machine elements experience during their service life due to varying operational conditions like speeds, weights, and dynamic forces.

Finite Life Design focuses on components expected to fail after a finite number of load cycles due to material fatigue. The design process uses S-N (Wöhler) curves to predict fatigue life by plotting stress amplitude against the number of cycles to failure. This approach is suitable for components subjected to high stresses or severe operating conditions with limited expected load cycles. Engineers ensure that the operational stress does not exceed the material's fatigue limit.

Infinite Life Design, on the other hand, applies to components expected to endure an indefinite number of cycles under variable loading. This philosophy is used for parts operating under low stress, where failure from fatigue is unlikely. Design ensures the stress remains below the material's endurance limit, enhancing durability. Examples include gears, shafts, and bearings under light loads. Strategies include reducing stress concentrations and selecting high-fatigue-strength materials.

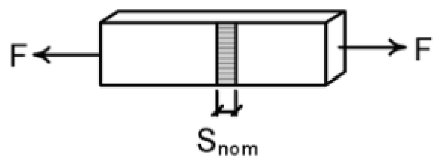
This distinction is vital in designing components for safety, reliability, and cost-effectiveness across different operational scenarios.

STRESS CONCENTRATION

In machine design, geometric irregularities like changes in cross sections, holes, grooves, and notches cause variations in stress distribution. These areas, known as **stress raisers**, lead to **stress concentrations**. Rotating shafts need shoulders for bearings and key slots for pulleys, while bolts have heads and threads that cause abrupt changes in cross sections. Stress concentrations can also arise from imperfections like tool marks or irregularities, which increase stress beyond what is predicted by basic stress equations.

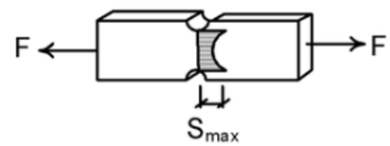
$$k_t = \frac{S_{max}}{S_{nom}}$$

Uniform bar in tension



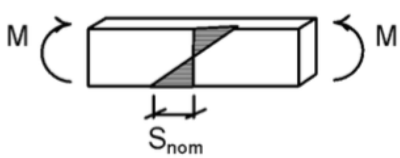
$$S_{nom} = \frac{F}{A}$$

Notched bar in tension



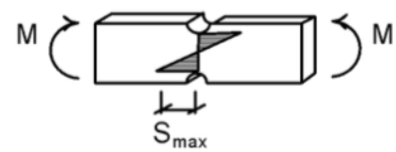
$$S_{max} = k_t \frac{F}{A}$$

Uniform bar in bending



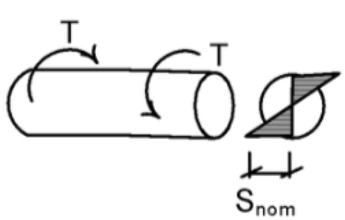
$$S_{nom} = \frac{Mc}{I}$$

Notched bar in bending



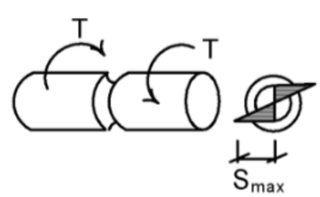
$$S_{max} = k_t \frac{Mc}{I}$$

Uniform shaft in torsion



$$S_{nom} = \frac{Tc}{J}$$

Notched shaft in torsion



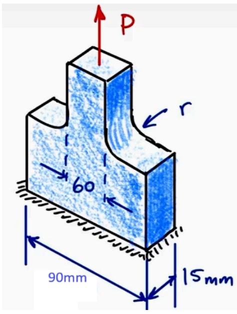
$$S_{max} = k_t \frac{Tc}{J}$$

Where: K_t = stress concentration factor
S_{nom} = nominal stress
Nominal stresses are the values that are calculated from direct or combined stress equations.
S_{max} = maximum stress
T = maximum torque
M = maximum moment

The regions where these stress increases occur are called **areas of stress concentration**, which are prone to failure if not carefully managed. Stress concentration is not due to the inherent properties of the material but rather due to changes in the geometry of the component. Examples include tool marks, key slots, or drilled holes.

Illustration:

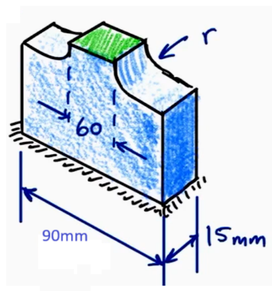
A stepped structural component is subjected to an axial load P. The material used has an maximum allowable stress, S_{max}= 125 MPa. Determine the maximum permissible load, P_{max} that can be applied to the structure without exceeding the allowable stress when r = 12mm and r = 18mm.



The maximum stress will occur at the smallest cross-sectional area perpendicular to the applied load, P.

Given: Maximum Allowable Stress (S_{max}): 125 MPa
Small width (d): 60 mm
Large width (D): 90 mm
Thickness (t): 15 mm
Ratio D/d: 90/60=1.5
Case 1 Fillet Radius (r_1): 12 mm→ Stress Concentration Factor (k_{t1}): 1.7
Case 2 Fillet Radius (r_2): 18 mm→ Stress Concentration Factor (k_{t2}): 1.5

Find: Critical Cross-sectional Area (A)
Maximum Load for r = 12 mm (P_{max1})
Maximum Load for r = 18 mm (P_{max2})



The cross-sectional area is calculated as:

$$A = (60mm)(15mm) = 900mm^2$$

The maximum stress at the critical section is given by:

$$S_{max} = k_t \frac{P_{max}}{A}$$

$$P_{max} = A \cdot \frac{S_{max}}{k_t}$$

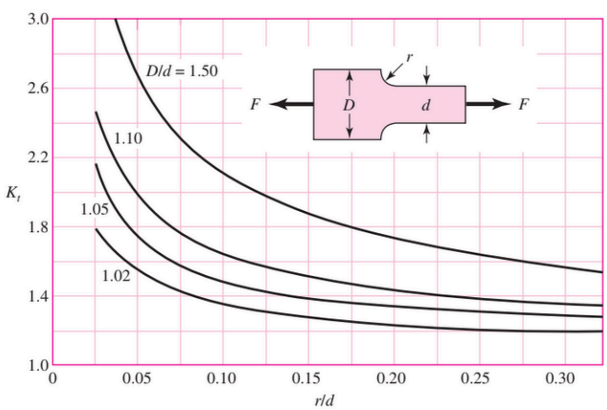
The k_t = Stress Concentration Factor, depends on fillet radius and geometry.

Shigley's Mechanical Engineering Design by Richard Budynas, Keith Nisbett

$$\frac{D}{d} = \frac{90mm}{60mm} = 1.5$$

Figure A–15–5

Rectangular filleted bar in tension or simple compression. $\sigma_0 = F/A$, where $A = dt$ and t is the thickness.



For fillet radius of r = 12mm, $\frac{r}{d} = \frac{12mm}{60mm} = 0.2$, the stress concentration factor, k_t = 1.7

$$P_{max} = A \cdot \frac{S_{max}}{k_t} = \left(900mm^2\right)\frac{125\,\frac{N}{mm^2}}{1.7} = 66,176.4706\,N$$

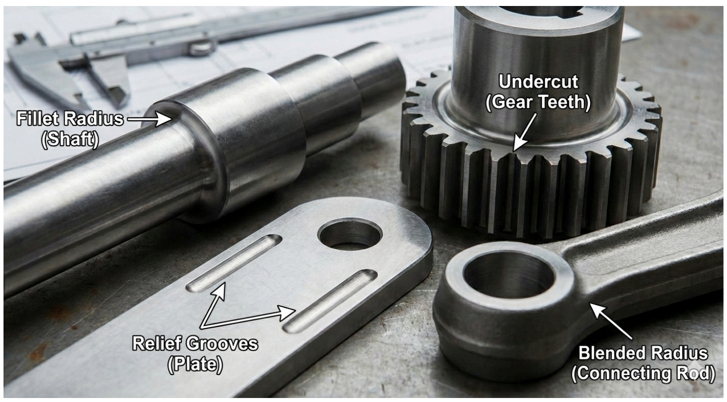
For fillet radius of r = 18mm, $\frac{r}{d} = \frac{18mm}{60mm} = 0.3$, the stress concentration factor, k_t = 1.5

$$P_{max} = A \cdot \frac{S_{max}}{k_t} = \left(900mm^2\right)\frac{125\,\frac{N}{mm^2}}{1.5} = 75,000.00\,N$$

Key Takeaways:

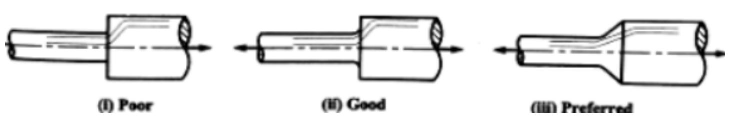
- The fillet radius directly affects the stress concentration factor (k_t).
- A larger fillet radius (r = 18) results in a lower k_t=1.5, which allows for a higher maximum permissible load (P_{max}=75,000 N).
- A smaller fillet radius (r = 12 mm) has a higher k_t=1.7, which reduces the maximum load capacity to 66,176.4706 N.
- Increasing the fillet radius helps reduce stress concentration, allowing for higher load capacity and improved structural integrity (a 13.3% increase).

Techniques for Reducing Stress Concentration



1. Use of Fillets

- Fillets are curved transitions added to sharp corners to smooth out stress flow lines.



  - (a) Poor: Sharp corners cause stress concentration.
  - (b) Good: A small fillet reduces the intensity of stress concentration.
  - (c) Preferred: Larger fillets offer the best reduction of stress.

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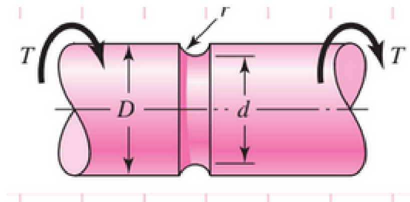
2. Gradual Change in Cross-Section

- Avoid abrupt changes in cross-sectional areas.
- Examples:
  - Tapered shafts instead of sharp shoulders.
  - Smooth blending between sections.

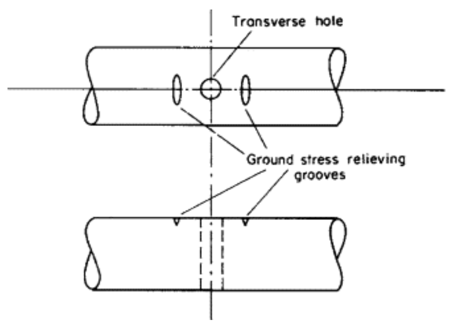
3. Stress Relievers in Shafts

- Shafts with shoulders can have stress reduced by using fillets.



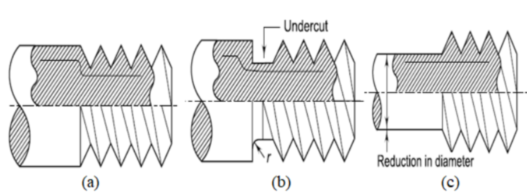
4. Holes and Notches in Members

- Stress around holes or notches can be reduced by:
  - Adding reinforcement in the form of washers or doublers around the holes.
  - Creating elliptical rather than circular holes to distribute stress more evenly.



In the design of machine shafts, geometric modifications such as fillets, notches, and grooves are used to control and direct the flow of stress within the material. For structural integrity, fillets and relief grooves are employed to guide stress lines away from critical corners, smoothing out the 'flow' to prevent cracks and strengthen the part. However, for safety purposes, we can also use this concept in reverse: by intentionally placing a sharp 'Shear Groove' or notch at a specific location, we direct the stress concentration to that single point. This ensures that if the machine is overloaded, the shaft will fail purely at this calculated 'fuse' point, protecting the more expensive components from damage. Thus, geometry is the tool we use to tell the stress exactly where to go—either to disperse it for strength or to concentrate it for safety.

5. Stress Concentration in Cylindrical Members with Holes



- (a) Poor: Abrupt transitions lead to significant stress concentration.
- (b) Good:A small fillet or curved transition is introduced at the junction between the smooth shaft and the threaded section.
- (c) Preferred: The transition between the smooth shaft and the threaded section is blended with a large fillet radius.

PROPERTIES OF SECTIONS: Design of Machine Elements 4th Edition by Virgil Faires

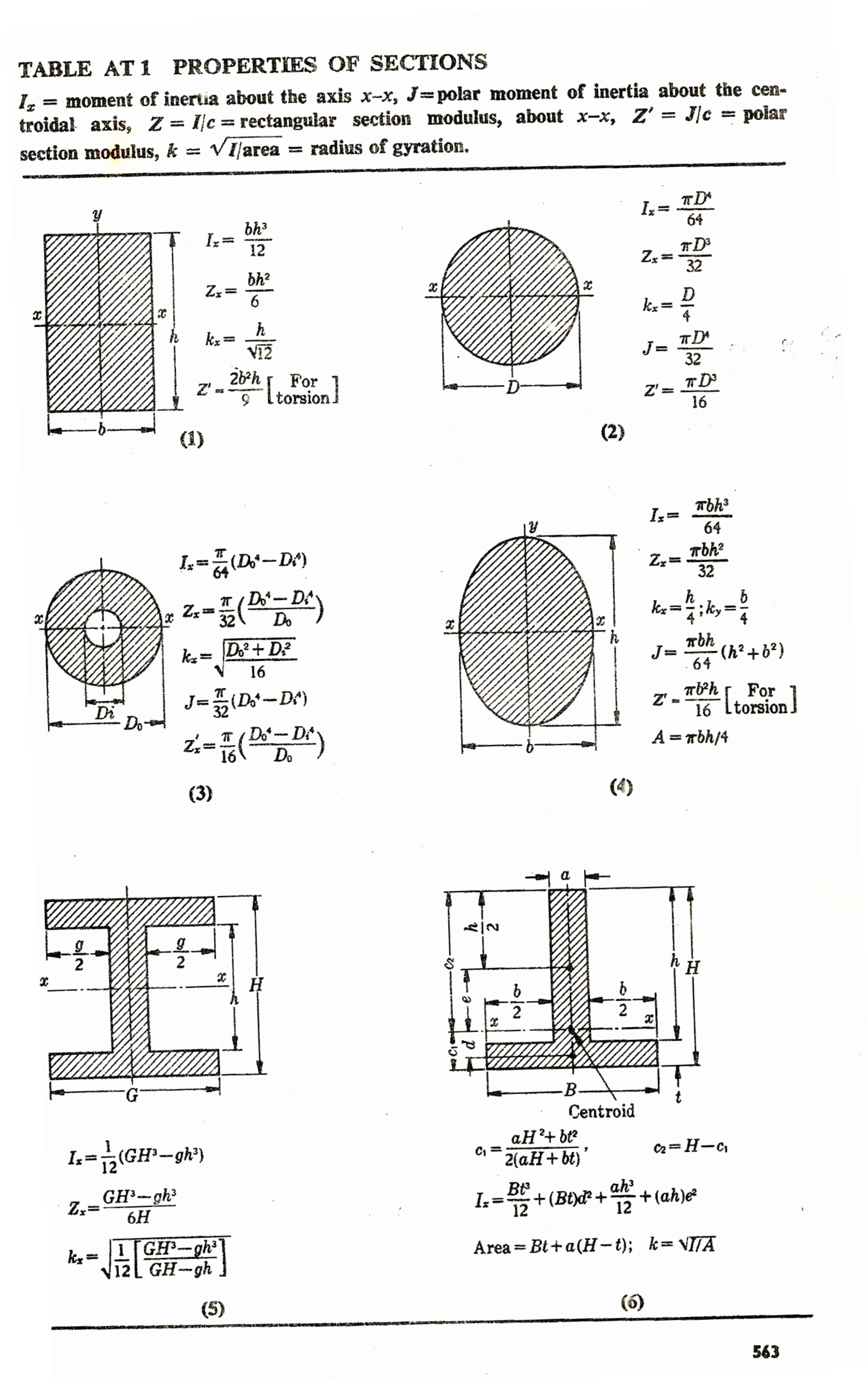


TABLE AT 1 PROPERTIES OF SECTIONS

I_x = moment of inertia about the axis x–x, J = polar moment of inertia about the centroidal axis, $Z = I/c$ = rectangular section modulus, about x–x, $Z' = J/c$ = polar section modulus, $k = \sqrt{I/\text{area}}$ = radius of gyration.

(1)

$I_x = \frac{bh^3}{12}$, $Z_x = \frac{bh^2}{6}$, $k_x = \frac{h}{\sqrt{12}}$, $Z' = \frac{2b^2h}{9}$ [For torsion]

(2)

$I_x = \frac{\pi D^4}{64}$, $Z_x = \frac{\pi D^3}{32}$, $k_x = \frac{D}{4}$, $J = \frac{\pi D^4}{32}$, $Z' = \frac{\pi D^3}{16}$

(3)

$I_x = \frac{\pi}{64}(D_o^4 - D_i^4)$, $Z_x = \frac{\pi}{32}\left(\frac{D_o^4 - D_i^4}{D_o}\right)$, $k_x = \sqrt{\frac{D_o^2 + D_i^2}{16}}$, $J = \frac{\pi}{32}(D_o^4 - D_i^4)$, $Z'_x = \frac{\pi}{16}\left(\frac{D_o^4 - D_i^4}{D_o}\right)$

(4)

$I_x = \frac{\pi b h^3}{64}$, $Z_x = \frac{\pi b h^2}{32}$, $k_x = \frac{h}{4}; k_y = \frac{b}{4}$, $J = \frac{\pi b h}{64}(h^2 + b^2)$, $Z' = \frac{\pi b^2 h}{16}$ [For torsion], $A = \pi bh/4$

(5)

$I_x = \frac{1}{12}(GH^3 - gh^3)$, $Z_x = \frac{GH^3 - gh^3}{6H}$, $k_x = \sqrt{\frac{1}{12}\left[\frac{GH^3 - gh^3}{GH - gh}\right]}$

(6)

Centroid

$c_1 = \frac{aH^2 + bt^2}{2(aH + bt)}$, $c_2 = H - c_1$

$I_x = \frac{Bt^3}{12} + (Bt)d^2 + \frac{ah^3}{12} + (ah)e^2$

Area $= Bt + a(H - t)$; $k = \sqrt{I/A}$

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