



### Defining the Timeline and Structure

- **Base Payment (A):** The maintenance cost this year (Year 1) is **₱5,200**.
- **Arithmetic Gradient (G):** The cost increases linearly by **₱300** each year for the "subsequent 9 years." This means the increases take place from Year 2 up to Year 10 (1 + 9 = 10 years total).
- **Interest Rate (i):** 16% compounded annually ( $i = 0.16$ ).
- **Total Lifespan (n):** The timeline spans a full **10-year period** ( $n = 10$ ).

Breaking down the complex cash flow into two independent components using the principle of superposition:

1. A uniform series (ordinary annuity) with a base value of  $A = \text{P}5,200$ .
2. An arithmetic gradient series with a constant step increase of  $G = \text{P}300$  starting at  $t=2$ .

- **Annuity Component ( $P_A$ ):**

$$P_A = \frac{A}{i} [1 - (1 + i)^{-n}] = \frac{\text{P}5,200}{0.16} [1 - (1 + 0.16)^{-10}] = \text{P}39,867.2171$$

- **Gradient Component ( $P_G$ ):**

$$P_G = \frac{G}{i} \left[ \frac{(1+i)^n - 1}{i(1+i)^n} - \frac{n}{(1+i)^n} \right] = \frac{\text{P}300}{0.16} \left[ \frac{(1.16)^{10} - 1}{0.16(1.16)^{10}} - \frac{10}{(1.16)^{10}} \right] = \text{P}4,811.98$$

- **Total Present Worth (P):**

$$P = P_A + P_G = \text{₱}39,867.2171 + \text{₱}4,811.98 = \text{₱}44,679.2011$$

In industrial asset management, a fixed annual maintenance cost is a myth. As machines, fleet vehicles, or chemical processing units age, their physical components suffer from friction, thermal fatigue, and general wear. This leads to predictably rising operation and maintenance (O&M) expenses.

The arithmetic gradient (G) perfectly models this real-world deterioration pattern. Recognizing that a machine will cost *₱300 more* to repair each subsequent year allows engineers to accurately calculate total life-cycle costs rather than underestimating costs with a flat average.