To evaluate a large power of the imaginary unit i, we rely on the predictable, repeating 4-step algebraic cycle generated by its sequential properties. This fundamental cycle is defined as follows:

- $i^1 = i$
- $i^2 = -1$
- $i^3 = -i$
- $i^4 = 1$

Any power of i higher than 4 simply repeats this exact same sequence. To solve i^{23} manually, we break the exponent down to isolate the highest multiple of 4 that fits inside 23, which is 20. Using the **Product Rule of Exponents**, we can partition the expression:

$$i^{23} = i^{20} \cdot i^3$$

Using the **Power Rule of Exponents**, we can rewrite i^{20} as $\left(i^4\right)^5$. Substituting our fundamental identity $i^4 = 1$ into this expression yields:

$$i^{23} = \left(i^4\right)^5 \cdot i^3 = (1)^5 \cdot i^3 = 1 \cdot i^3 = i^3$$

Since the expression simplifies completely to i^3, we refer back to our core cyclic definitions where $i^3 = -i$. This confirms that i^{23} is equivalent to $-i$.

CALCULATOR TIPS:

Method 1:

If you are using a modern scientific calculator like the **Casio fx-991EX ClassWiz**, it can compute high imaginary cycles directly without giving a mathematical breakdown.

1. Shift your calculator into Complex Mode by pressing **MENU 2**.
2. Type out the expression exactly using the imaginary key: i^23 (The i button is located above the ENG key).
3. Press =, and the screen will directly display $-i$.

Crucial Exam Warning for older Casio models (fx-991ES Plus): If you try to type i^23 in Complex Mode (MODE 2) on an older ES model calculator, the screen will flash a Math ERROR. This occurs because the older processing chip can only handle manual complex squares (i^2) or cubes (i^3) directly and fails on higher exponents. If your calculator throws an error, use Method 2 instead.

Method 2: The Core Remainder Rule (For All Calculators)

Because the powers of i reset completely every 4 steps, the value of any exponent depends strictly on its arithmetic **remainder** when divided by 4.

1. Take the given exponent and divide it by 4 on your standard calculation screen:

$$\frac{23}{4} = 5.75$$

2. Look exclusively at the **decimal portion** of your answer to determine the cyclic remainder:
   - **.25** ⇒ Remainder of 1 ⇒ i
   - **.50** ⇒ Remainder of 2 ⇒ -1
   - **.75** ⇒ Remainder of 3 ⇒ -i
   - **.00** ⇒ Remainder of 0 ⇒ 1

3. Since $\frac{23}{4}$ gives a decimal ending in **.75**, it tells you instantly that it lands perfectly on the third step of the cycle, which is $-\ i$.