To solve the vector expression $\left(A_1 + A_2\right) \cdot A_1$, we must follow the standard order of operations for vector arithmetic, beginning with the addition inside the parentheses followed by the dot product (scalar product).

1. **Vector Addition $\left(A_1 + A_2\right)$:** When adding two vectors, we independently combine their respective orthogonal component coefficients $(i,\ j,\ k)$:

$$A_1 = 3i - 3j + 7k$$

$$A_2 = 9i - 3j - 4k$$

$$\left(A_1 + A_2\right) = (3 + 9)i + (-3 + 3)j + (7 - 4)k$$

$$\left(A_1 + A_2\right) = 12i + 0j + 3k$$

2. **The Dot Product $\left[\left(A_1 + A_2\right) \cdot A_1\right]$:**
   The dot product multiplies the corresponding components of the two vectors together and sums the results to yield a single scalar value. We take our newly calculated sum vector and dot it with vector A_3:

$$A_3 = 2i - 5j - 2k$$

$$\left(A_1 + A_2\right) \cdot A_1 = (12)(2) + (0)(-5) + (3)(-2)$$

$$\left(A_1 + A_2\right) \cdot A_1 = 24 + 0 - 6 = 18$$