To find the original function $h(x)$, we must analyze the mechanics of **composite functions**. A composite function, written as $h[f(x)]$, represents a multi-stage operation where the output of the inner function $f(x)$ is used directly as the input for the outer function $h(x)$.

We are given two vital pieces of information:

1. The composite result: $h[f(x)] = \left(2x^2 - 1\right)^5$
2. The inner component: $f(x) = 2x^2 - 1$

To isolate the structural rule for $h(x)$, we can use an algebraic technique known as **variable substitution**. Let us temporarily define a new placeholder variable, u, to represent the entire inner function expression. Therefore, we let:

$$u = 2x^2 - 1$$

Now, we substitute this placeholder variable u into our composite equation wherever the group $\left(2x^2 - 5\right)$ appears:

$$h[f(x)] = \left(2x^2 - 1\right)^5 \Rightarrow h(u) = u^5$$

Because the variable name used as an input placeholder does not alter the mathematical behavior of the function structure, we can safely replace the dummy variable u back with the standard independent variable x. This reveals our clean, fundamental outer function: $h(x) = x^5$.