

To determine the value of the unknown coefficient m such that the quadratic equation $9x^2 + mx + 1 = 0$ yields exactly one real solution, we must utilize the mathematical properties of the **discriminant**. In any standard quadratic equation of the form $Ax^2 + Bx + C$, the nature of its roots (solutions) is completely dictated by the radicand found underneath the radical sign in the quadratic formula, known specifically as the discriminant: $D = B^2 - 4AC$.

For a quadratic equation to possess exactly one unique, repeating real solution (geometrically meaning its parabolic graph is perfectly tangent to the x-axis at a single vertex point), the discriminant must be exactly equal to zero ($D = 0$). First, we map out our specific coefficients from the given problem: $A = 9$, $B = m$ and $C = 1$. Substituting these parameters directly into our discriminant expression establishes the following relationship:

$$D = m^2 - 4(9)(1) = 0$$

Now, we perform basic algebraic manipulation to isolate and solve for our target variable:

$$m^2 - 36 = 0$$

$$m^2 = 36$$

Taking the square root of both sides of the equation releases the variable from its power state. Because squaring both a positive and a negative value yields an identical positive number, we must account for both algebraic directions:

$$m = \pm \sqrt{36} = \pm 6$$

The Perfect Square Trinomial Rule

On a conceptual level, any quadratic equation with exactly one real solution forms a **Perfect Square Trinomial** when its coefficients are balanced. You can quickly remember that the middle coefficient B is always equal to twice the product of the square roots of the outer coefficients A and C :

$$B = \pm 2\sqrt{A \cdot C}$$

$$m = \pm 2\sqrt{9 \cdot 1} = \pm 2(3) = \pm 6$$