

In mathematics and engineering, a complex number can be expressed in three primary formats: **Rectangular form** ($z = a + bi$), **Polar form** ($z = r \angle \theta$), and **Exponential form** ($z = re^{i\theta}$).

r represents the exact straight-line distance from the origin to the point on the complex plane (the magnitude), and θ represents the angle of rotation from the positive real axis. While pure mathematics strictly requires θ to be written in radians for calculus applications, engineering board exams frequently present θ in **degrees** inside the exponential format for quick practical reading.

To convert the rectangular complex number $1 + 5i$ into its exponential counterpart, we need to calculate two independent values: the magnitude (r) and the angle (θ).

1. **Calculate the Magnitude (r):**

Using the Pythagorean theorem on the real component ($a = 1$) and the imaginary component ($b = 5$):

$$r = \sqrt{a^2 + b^2} = \sqrt{1^2 + 5^2} \approx 5.1$$

2. **Calculate the Phase Angle (θ):**

Using the inverse tangent function to find the direction of the vector:

$$\theta = \arctan\left(\frac{b}{a}\right) = \arctan\left(\frac{5}{1}\right) \approx 78.69^\circ$$

3. **Assemble the Exponential Form:**

Plugging our calculated values back into the $re^{i\theta}$ model yields:

$$z = 5.1e^{78.69^\circ i}$$