Partial Fraction Decomposition is the reverse process of finding a common denominator to combine rational fractions. When you add fractions together, you combine them into a single, complex rational expression. Decomposing into partial fractions means taking that single, complex fraction and breaking it down into a sum of simpler, "partial" fractions whose denominators are the individual linear or quadratic factors of the original denominator.

This technique is incredibly vital in calculus for solving complex integrals, and in engineering for finding inverse Laplace transforms.

When the denominator consists of a linear factor raised to a power, such as $(x - 2)^3$, it represents a **repeated linear factor**. The rules of partial fractions dictate that we must set up a separate fraction for every increasing power of that factor up to its maximum exponent:

$$\frac{3x^2-8x+9}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$$

The Shortcut Method (Synthetic Substitution / Variable Shift)

For fractions where the denominator is entirely a single repeated linear factor like $(x - 2)^3$, there is a rapid algebraic trick that completely skips systems of equations. Let a new variable $u = x - 2$. This means $x = u + 2$. Substitute this directly into the numerator:

1. Substitute $x = u + 2$ into the numerator $3x^2 - 8x + 9$:

   $$3(u + 2)^2 - 8(u + 2) + 9$$

2. Expand it out:

   $$3(u^2 + 4u + 4) - 8u - 16 + 9$$

   $$3u^2 + 12u + 12 - 8u - 16 + 9$$

3. Combine like terms:

   $$3u^2 + 4u + 5$$

4. Put it back over the original denominator (u^3):

   $$\frac{3u^2+4u+5}{u^3} = \frac{3u^2}{u^3} + \frac{4u}{u^3} + \frac{5}{u^3}$$

   $$= \frac{3}{u} + \frac{4}{u^2} + \frac{5}{u^3}$$

5. Replace u back with $(x - 2)$:

$$\frac{3x^2-8x+9}{(x-2)^3} = \frac{3}{(x-2)} + \frac{4}{(x-2)^2} + \frac{5}{(x-2)^3}$$

Board Exam Quick Tips & Calculator Guide

Partial fraction problems on the board exam should **never** be solved using long hand math. Since you are given multiple choices, you can use a flawless numerical verification technique called the **CALC Trick (Reverse Engineering)**.

The Equation Substitution Test (CALC Trick)

If two algebraic expressions are truly identical, they must yield the exact same numerical output when evaluated at any arbitrary value of x.

1. Look at the original problem fraction: $\frac{3x^2-8x+9}{(x-2)^3}$

2. Pick an easy, random test value for x that doesn't cause a zero denominator. Let's choose **x = 5**.

3. Plug x = 5 into the original expression using your calculator:

$$\frac{3(5)^2-8(5)+9}{(5-2)^3} \approx 1.6296$$

4. Now, test your multiple choice options by typing them into your calculator and hitting **CALC**, entering 5.

5. Let's look closely at option (d) but applying the corrected denominator matching the problem:
    - Type 3 / (X - 2) + 4 / (X - 2) ² + 5 / (X - 2) ³ into your main screen.
    - Press **CALC**, type 5, and hit **=**.
    - The calculator screen will output $\frac{44}{27}$ $(or\ 1.6296)$.

Because the numbers match perfectly, you can confidently encircle option **d** knowing that it is structurally the correct answer.