In algebra and engineering board exams, number relation problems often expand beyond simple linear equations into **non-linear systems**. These problems give conditions involving basic sums alongside higher-degree combinations, such as the sum of squares ($x^2 + y^2$) or the product of the terms ($xy$).

Solving these problems systematically requires identifying the relationship between basic algebraic binomial identities and their component parts.

*Thrice the sum of two numbers is 33. The sum of the squares of two numbers is 61. Find the product of the two numbers.*

**<u>Step 1:</u> Define the Variable Equations**

Let the first number be x and the second number be y.

- **Condition 1:** *"Thrice (3 times) the sum of two numbers is 33"*:

$$3(x + y) = 33$$

$$x + y = 11$$

- **Condition 2:** *"The sum of the squares of the two numbers is 61"*:

$$x^2 + y^2 = 61$$

Find the value of the product xy.

**<u>Step 2:</u> Isolate a Single Variable**

From Condition 1, express x in terms y:

$$x = 11 - y$$

**<u>Step 3:</u> Substitute into Condition 2**

Substitute this expression into the sum of squares equation:

$$(11 - y)^2 + y^2 = 61$$

Expand the perfect square binomial:

$$y^2 - 11y + 30 = 0$$

**<u>Step 4:</u> Solve the Quadratic Equation**

$$(y - 6)(y - 5) = 0$$

This yields two valid values for y:

$y_1 = 6$ and $y_2 = 5$

**<u>Step 5:</u> Determine the Final Product**

Using x = 11 - y:

- If y = 6 ⇒ x = 11 - 6 = 5
- If y = 5 ⇒ x = 11 - 5 = 6

Therefore, the two numbers are **5 and 6**. Their product is:

$$xy = (5)(6) = 30$$