

Work problems model the efficiency of individuals, machines, or systems performing a task either independently or collaboratively.

The structural foundation of work problems relies on the concept of **work rate (R)**, which represents the fraction of a total job completed per unit of time (T):

$$\text{Rate (R)} = \frac{\text{Total Work Completed (W)}}{\text{Time Spent (T)}}$$

When evaluating a single standard project, we set the total number of job units (W) equal to 1 (signifying 100% completion of one full task). Therefore, the individual rate of any worker is simply the reciprocal of their total independent completion time:

$$R = \frac{1}{T}$$

When multiple entities work simultaneously without interfering with one another, their individual working rates add together directly to form a combined net rate (R_{total}):

$$R_1 + R_2 = R_{\text{total}}$$

$$\frac{1}{T_1} + \frac{1}{T_2} = \frac{1}{T_{\text{together}}}$$

X and Y working together can finish painting a home in 7 days. X, working alone, can finish it in 4 days less than Y. How long will it take each of them to finish working alone?

- **Combined Time (T_{together}):** 7days
- **Worker X's Time (x):** Finishes in 4 days less than Y ($x = y - 4$)
- **Worker Y's Time (y):** Base variable

Step 1: Set Up the Work Rate Reciprocal Equation

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{7}$$

Substitute the relationship $x = y - 4$ into the formula to isolate a single variable:

$$\frac{1}{y-4} + \frac{1}{y} = \frac{1}{7}$$

$$y^2 - 18y + 28 = 0$$

Step 2: Solve Using the Quadratic Formula

Because this equation cannot be factored into clean whole integers, use the quadratic formula. Solve this directly using your calculator. This gives two possible mathematical roots:

$$y_1 = 16.2801 \text{ days}$$

$$y_2 = 1.7199 \text{ days}$$

Step 3: Verify and Round the Results

- **Analyze $y_2 = 1.7199$ days:** Disregard this root. If Y took only 1.72 days, then X would take $1.72 - 4 = -2.28$ days, which is physically impossible.
- **Analyze $y_1 = 16.2801$ days:** This is our valid baseline time for worker Y. Rounding to the nearest integer option gives **16 days**.
- **Find X's Time:** $x = 16.2801 - 4 = 12.2801$ days. Rounding to the nearest integer option gives **12 days**.