In algebra and engineering physics, advanced motion problems frequently involve an object moving through a fluid medium that is also in motion, such as an airplane flying through moving air (wind) or a boat traveling through flowing water (river current).

These scenarios are governed by the principles of **Relative Velocity Vector Addition**. The core concept dictates that the actual, observed speed of the object relative to the ground (called the ground speed, v_g) is the algebraic sum of the object's independent velocity in stationary fluid (still air speed, v) and the velocity of the medium itself (wind speed, x).

Headwinds vs. Tailwinds

When handling aircraft motion along a single axis, the direction of the wind relative to the plane dictates the algebraic sign used to model the system's true ground speed:

1. Flying Against the Wind (Headwind)

When a plane flies directly into a headwind, the moving air opposes its forward progress. This retarding force slows the plane down relative to the ground. Structurally, the wind velocity is subtracted from the plane's still-air velocity:

$$Net\ Speed\ \left(v_1\right) = v - x$$

2. Flying With the Wind (Tailwind)

When a plane flies in the same direction as the wind, it experiences a tailwind. The moving air pushes the aircraft forward, accelerating its progress relative to the ground. Structurally, the wind velocity adds directly to the plane's still-air velocity:

$$Net\ Speed\ \left(v_2\right) = v + x$$

A plane takes the same amount of time to fly 40 miles against the wind as it does to fly 50 miles with the wind. If the speed of the wind is 20 mph, determine the speed of the plane in still air.

<u>Step 1:</u> Isolate the Given Parameters

- Distance against the wind (s_1) = 40 miles
- Distance with the wind (s_2) = 50 miles
- Velocity of the wind (x) = 20 mph

Unknown: Velocity of the plane in still air = v

<u>Step 2</u>: Establish the Time Invariant Constraint

The critical sentence that links both states together is: *"A plane takes the* ***same amount of time****..."* This means that the time spent on the headwind leg (t_1) is exactly equal to the time spent on the tailwind leg (t_2):

$t_1 = t_2$

<u>Step 3:</u> Set Up the Proportional System

Since the standard equation for motion is $s = vt$, isolating time yields $t = \frac{s}{v}$. Substituting this definition into our time invariant constraint produces a ratio equation:

$$\frac{s_1}{v_1} = \frac{s_2}{v_2}$$

$$\frac{s_1}{v - x} = \frac{s_2}{v + x}$$

Step 4: Substitute and Solve for v

Plugging our known values into the ratio yields:

$$\frac{40}{v - 20} = \frac{50}{v + 20}$$

$$v = 180\ mph$$