Variation problems describe these relationships using a constant of proportionality, denoted as k.

There are three primary categories of variation:

1. **Direct Variation ($x \propto y$):** As y increases, x increases proportionally (x = ky).
2. **Inverse Variation $\left(x \propto \frac{1}{z}\right)$:** As z increases, x decreases proportionally $\left(x = \frac{k}{z}\right)$.
3. **Joint/Combined Variation:** Occurs when a variable depends on two or more variables at once. In this problem, x varies **directly** as y and **inversely** as z which mathematically translates to:

$$x = k\left(\frac{y}{z}\right)$$

A standard textbook solution involves a two-stage process: first, solve for the unique variation constant (k) using the initial conditions, then use that constant to find the unknown value.

If x varies directly as y and inversely as z, and x=18 when y=8 and z=3, find the value of x when y=15 and z=5.

Step 1: Solve for the Proportionality Constant (k)

Substitute values of x, y and z to the governing equation:

$$18 = k\left(\frac{8}{3}\right)$$

Solve for k; $k = \frac{54}{8} = 6.75$

Stage 2: Calculate the New Value of x

Now, find x when y = 15 and z = 5, maintaining our constant value k = 6.75:

$$x = 6.75\left(\frac{15}{5}\right)$$

Simplify the fraction inside the parentheses:

$$x = 20.25 = 20\frac{1}{4}$$