# WORDED PROBLEMS

## NUMBER PROBLEM

**Number problems** serve as the foundational entry point for translating word-based descriptions into precise algebraic equations. These problems test an engineer's ability to interpret structural criteria—such as sequential continuity, digit positioning, or mathematical proportions—and map them into deterministic systems of linear equations.

Mastering number problems requires a solid understanding of how numerical expressions are mathematically structured and how constraints like "consecutive," "exceeds," or "reciprocal" function as algebraic operators.

### Consecutive Integers

A high-frequency subset of number problems involves **consecutive integers** (whole numbers that follow each other in order without gaps, from smallest to largest).

### Standard Consecutive Integers

When numbers follow each other sequentially with a constant step-size of exactly 1 (e.g.,14, 15, 16), we assign the initial base integer as our primary independent variable x. Each subsequent element scales upward by adding 1:

- First Integer: x
- Second Integer: x + 1
- Third Integer: x + 2

### Consecutive Even or Consecutive Odd Integers

A common area of confusion is setting up expressions for consecutive **even** numbers (e.g., 2, 4, 6) versus consecutive **odd** numbers (e.g., 11, 13, 15).

Whether integers are strictly even or strictly odd, they always sit exactly **two units apart** from one another on a standard number line. Therefore, the algebraic representations for consecutive even numbers and consecutive odd numbers are completely identical!

- First Even/Odd Integer: x
- Second Even/Odd Integer: x + 2
- Third Even/Odd Integer: x + 4

The only mathematical differentiator between the two configurations is the constraint placed on the starting value x: if the problem dictates odd integers, the resulting baseline value of x will automatically calculate to be an odd value.

**Example:**

*"Find 3 consecutive odd integers whose sum is 39."*

**Step 1:** **Define the Variables:** Let the three consecutive odd integers be defined relative to our baseline variable x:

$$First\ integer = x$$
$$Second\ integer = x + 2$$
$$Third\ integer = x + 4$$

**Step 2:** **Translate the Condition into an Equation:** The word "sum" requires adding all three components together, and "is" serves as our structural equals operator (=):

$$x + (x + 2) + (x + 4) = 39$$

$$3x + 6 = 39$$

$$x = 11$$

**Step 3:** **Generate the Complete Solution Set:** Now that we have established x = 11, evaluate the remaining integer placeholders:

- First Integer: x = 11
- Second Integer: 11 + 2 = 13
- Third Integer: 11 + 4 = 13

The three consecutive odd integers are **11, 13, and 15**.

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# AGE PROBLEM

### 1. The Core Governing Principle

The time elapsed for all individuals involved is equal.

If five years pass for one person, exactly five years must pass for another person. When moving backward or forward through time, you must apply the exact same mathematical addition or subtraction across all variable fields representing the individuals involved.

## 2. The Matrix Construction Method

The most practical method to prevent structural errors in word problems is to construct an Age Matrix Table. This table maps the individuals involved along one axis and the distinct time periods (Past, Present, or Future) along the other.

**Example:**

> *"A father is three times as old as his son. Five years ago, he was five times as old as his son was at that time. How old is his son?"*

**Step 1:** Define the Present Variables

Always baseline your equations from the most logical reference variable—typically the younger individual's current status.

- Let the present age of the Son = $x$
- Since the father is currently three times as old as the son, the present age of the Father = $3x$

**Step 2:** Shift to the Past Time Horizon
The second statement introduces a time jump: *"Five years ago"*. According to our core principle, we subtract exactly $5$ from both individuals' present ages:

- Son's Past Age: $x - 5$
- Father's Past Age: $3x - 5$

This allows us to complete our structural age matrix table:

| | **Past (5 years ago)** | **Present** |
|---|---|---|
| Father | $3x - 5$ | $3x$ |
| Son | $x - 5$ | $x$ |

**Step 3:** Establish the Equation Relation

Now, translate the remaining relational sentence into an algebraic balance: *"...he [the father] was five times as old as his son was at that time."*

$$Father's\ Past\ Age = 5(Son's\ Past\ Age)$$

$$3x - 5 = 5(x - 5)$$

**Step 4:** Solve Using Algebraic Principles

Using the Distributive Property, expand the right-hand side of the equation:

$$3x - 5 = 5x - 25$$

Isolate the variable $x$ by shifting all terms containing variables to one side and constants to the other and solve for $x$,

$$x = 10$$

The present age of the son is 10 years old. Consequently, the father's current age is $3(10) = 30$ years old.

**Board Examination Optimization Tips:**

Because age problems must yield a precise real-number match, you can skip full algebraic derivations by testing the provided multiple-choice options inside your constructed matrix table. If an option does not satisfy both the present and past conditions simultaneously, it can be instantly eliminated.

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# MIXTURE PROBLEM

### 1. The Core Governing Principle

The foundational rule governing all algebraic mixture problems is the **Law of Conservation of Mass**. The total mass or volume of a specific pure substance before mixing must exactly equal the total mass or volume of that pure substance after mixing.

Amount of Pure Substance:

$$Amount\ of\ pure\ substance = [\%Decimal\ concentration][Total\ weight\ or\ volume\ of\ mixture]$$

Continuity Balance:

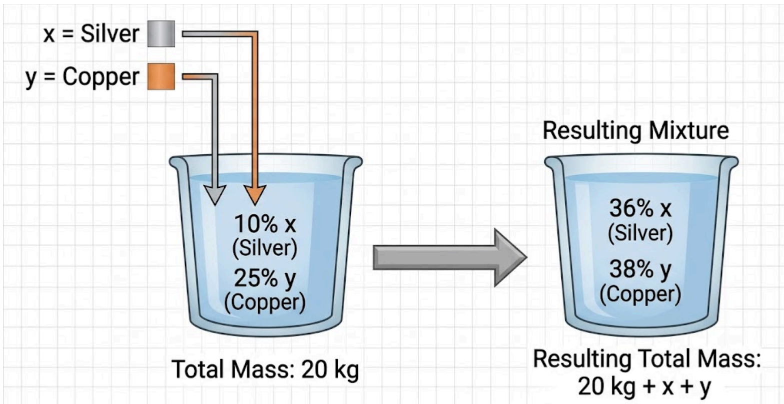
$$[Amount\ of\ substance\ in\ original\ mixture] \pm [Amount\ of\ substance\ added/subtracted] = [Amount\ of\ substance\ in\ resulting\ mixture]$$

### 2. Setting Up Simultaneous Equations

When a problem contains **two unknown quantities**, you must construct a system of linear equations. One equation tracks the *total aggregate mass* of the system, while the other tracks the *mass of a specific constituent element*.

**Example:**

*"How much pure silver and pure copper must be added to a 20 kg alloy containing 10% silver and 25% copper to produce a new resulting alloy containing 36% silver and 38% copper?"*




## Step 1: Define Your Variable States

- Let $x = mass\ of\ pure\ silver\ to\ be\ added\ (in\ kg)$
- Let $y = mass\ of\ pure\ copper\ to\ be\ added\ (in\ kg$
- **Total weight of the final mixture:** The original weight was 20 kg. After adding $x$ kg of silver and $y$ kg of copper, the final weight becomes exactly $(20 + x + y)$ kg.

## Step 2: Establish the Continuity Balance for Silver

Pure added silver $(x)$ has a silver concentration of 100% (or 1.0 as a decimal fraction).

$$Initial\ Silver + Added\ Silver = Final\ Silver$$

$$(0.10)(20) + (1.0)(x) = (0.36)(20 + x + y)$$

Let's simplify this equation completely:

$$2 + x = 7.2 + 0.36x + 0.36y$$

$$0.64x - 0.36y = 5.2 \qquad (eq.\ a)$$

## Step 3: Establish the Continuity Balance for Copper

Pure added copper $(y)$ has a copper concentration of 100 (or 1.0 as decimal fraction).

$$Initial\ Copper + Added\ Copper = Final\ Copper$$

$$(0.25)(20) + (1.0)(y) = (0.38)(20 + x + y)$$

Let's simplify this equation completely:

$$5 + y = 7.6 + 0.38x + 0.38y$$

$$-0.38y + 0.62y = 2.6 \qquad (eq.\ b)$$

**Step 4: Solve the Simultaneous System**

Solving **Equation A** and **Equation B** simultaneously yields the following results:

- $x = 16\ kg$ **of pure Silver**
- $y = 14\ kg$ **of pure Copper**

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## CLOCK PROBLEM

These problems model the relative motion of the two moving components of an analog clock: the **minute hand** and the **hour hand**. Because both hands move continuously at different, fixed uniform speeds, clock problems are fundamentally rate-time-distance problems applied over a circular geometry.

The **Universal** $\frac{12}{11}$ **Shortcut Rule** is a powerful mathematical trick used to solve clock tracking problems in under 10 seconds. In engineering board exams, these problems typically ask things like: *"At what time after 3 o'clock will the hands of a clock be together / perpendicular / opposite each other?"*


**The Core Governing Principle**

A standard clock face is divided into 60 minute spaces:

1. **The Minute Hand:** Travels at a rate of exactly **1 minute space per minute**.

2. **The Hour Hand:** Travels much slower. It takes a full hour (60 minutes) to move across just 5 minute spaces (for example, moving from the 12 to the 1 mark). Therefore, its rate is $\frac{5}{60} = \frac{1}{12}$ **minute spaces per minute**.

Because both hands move simultaneously in the same clockwise direction, the minute hand must constantly play catch-up. The relative speed at which the minute hand gains ground on the hour hand is the difference between their individual velocities:

$$Relative\ speed\ = 1 - \frac{1}{12} = \frac{11}{12}\ minutes\ spaces\ per\ minute$$

**Deriving the Shortcut Formula**

According to the laws of motion, the time (T) required to cover a certain distance at a given relative speed is:

$$Time\ = \frac{Distance\ needed\ to\ cover}{Relative\ speed}$$

If we substitute our relative speed into this equation:

$$Time\ = \frac{Initial\ Minute\ Spaces}{\frac{11}{12}}$$

When you divide by a fraction, you multiply by its reciprocal. This isolates our core shortcut parameter:

$$Time\ (Minutes)\ = X\frac{11}{12}$$

Where **X** is defined as the **"Initial Minute Spaces"** the minute hand must travel to reach the target condition, *assuming the hour hand stays completely frozen at the starting hour mark.*

**Example:**

*"At what time between 4 and 5 o'clock does the minute hand of a clock coincide with each other?"*

**Step 1:** Freeze the hour hand directly on the **4** mark.

**Step 2:** To be together, the minute hand must travel from the 12 mark directly to the 4 mark. Looking at a clock face, the 4 mark corresponds exactly to **20 minutes**. So, X = 20.

**Step 3:** Apply the multiplier:

$$Minutes\ = (20)\left(\frac{11}{12}\right) = \frac{240}{11} = 21.82\ minutes$$

The hands will align at exactly **4:21.82**.

# VARIATION PROBLEM

**Variation** describes the structural relationship between two or more interacting variables. It quantifies how a change in one quantity systematically influences another. Variation problems form the foundation for modeling physical laws, balancing economic scales, and solving engineering design constraints.

There are three primary types of mathematical variations: **Direct**, **Inverse**, and **Joint** (along with Combined variation, which merges these concepts). Every variation problem relies on a constant scaling factor denoted as $k$, known explicitly as the **constant of variation** or the **constant of proportionality**. This constant serves as the mathematical link that converts a proportional statement ($\alpha$) into a functional, working equation ($=$).

### 1. Direct Variation

A **direct variation** describes a relationship where two variables move in the exact same direction. If an independent variable increases, the dependent variable increases proportionally. Conversely, if the independent variable decreases, the dependent variable decreases. Geometrically, a direct variation equation forms a straight line passing perfectly through the origin (0,0), where the constant $k$ represents the constant slope of that line.

The proportional statement "x varies directly as y" is written as:

$$x \, \alpha \, y$$

By introducing the constant of proportionality $k$, we transform this relationship into a linear algebraic equation:

$$x = ky$$

To find the constant of variation $k$, the equation can be rearranged into a fixed ratio:

$$k = \frac{x}{y}$$

**Example**

- **Hooke's Law:** The force (F) required to compress or extend a mechanical spring is directly proportional to the displacement distance (x) it stretches. This is modeled as F=ks, where k is the specific spring constant.

### 2. Inverse Variation

An **inverse variation** describes an opposite relationship between two variables. When one variable increases, the other variable decreases at a proportional rate, and vice versa. Geometrically, the graph of an inverse variation relationship does not form a straight line; instead, it traces a curve known as a **hyperbola**. As one variable approaches infinity, the other asymptotes toward zero, meaning neither variable can ever equal zero.

The proportional statement "x varies inversely as y" is written as:

$$x \,\alpha\, \frac{1}{y}$$

Converting this relationship into an algebraic equation using the constant $k$ yields:

$$x = k\frac{1}{y}$$

To isolate the constant of variation $k$, we observe that the product of the two variables must always remain perfectly constant:

$$k = x \cdot y$$

**Example**

- **Boyle's Law:** In thermodynamics, the absolute pressure (P) exerted by a given mass of an ideal gas is inversely proportional to the volume (V) it occupies, provided the temperature remains constant. This is modeled as $P = \frac{k}{V}\ or\ PV = k$..

### 3. Joint Variation

A **joint variation** occurs when a dependent variable depends simultaneously on the direct product of two or more independent variables. It behaves exactly like a direct variation, but handles multiple inputs at the same time. If any of the independent variables increase, the dependent variable increases as well.

The proportional statement "x varies jointly as y and z" is modeled algebraically as:

$$x = k\, yz$$

To isolate the constant of variation k, we divide the dependent variable by the product of the independent variables:

$$k = \frac{x}{yz}$$

**Example**

- **Electrical Power:** The electric power (P) dissipated by a pure resistive circuit varies jointly as the electrical resistance (R) and the square of the current ($I^2$). This is modeled via Joule's Law as $P = kI^2$ (where k=1 in standard SI units).

**4. Combined Variation**

A **combined variation** describes a relationship that merges two or more different types of variations at the same time—specifically, a mix of **direct variation (multiplication)** and **inverse variation (division)**.

When a variable x varies directly as y and inversely as z, we blend the two foundational rules together into a unified expression:

$$x = k\frac{y}{z}$$

To isolate the constant of variation $k$ for a combined system, we rearrange the terms:

$$k = \frac{xz}{y}$$

**Example**

- **The Ideal Gas Law:** The volume (V) of a gas varies directly with its absolute temperature (T) and inversely with its pressure (P). This combined relationship is modeled as $V = \frac{kT}{P}$, which rearranges into the classical thermodynamic equation, $PV = nRT$.

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# WORK PROBLEM

**Work problems** model scenarios involving one or more entities performing a specific task at a given efficiency over time. These entities can be human laborers, mechanical components, or fluid systems such as inlet and drain pipes.

The entire framework of work problems rests on a fundamental law of physics: **the amount of work completed is equal to the rate of work multiplied by the time spent working**.

$$Work\ Done\ (W) = Rate\ (R) \cdot Time\ (T)$$

When a problem states that a task is completed—such as filling a single tank, painting a house, or paving a road—the total number of completed job units ($W$) is mathematically defined as exactly $1$. Therefore, for a single entity working to complete a job alone, the equation simplifies to:

$$RT = 1$$

$$R = \frac{1}{T}$$

This reveals a critical principle: **an entity's individual rate of work is the reciprocal of the total time it takes for that entity to finish the job entirely on its own**. For instance, if an input pipe requires 9 hours to fill a tank alone, its rate of performance is precisely $\frac{1}{9}$ of the tank per hour.

**Scenarios:**

Work problem variations are classified by how the individual working entities interact.

**1. Multiple Workers Operating at Different Rates and Durations**

When several distinct workers perform a task independently for varying lengths of time, their individual contributions accumulate linearly to finish the task:

$$R_1T_1 + R_2T_2 + \ ...\ + R_nT_n = 1$$

**2. Multiple Workers Operating Simultaneously (Together)**

If a group of workers or systems perform a task cooperatively, starting and stopping at the exact same time, they share a common total elapsed time variable (T). Because their efforts occur concurrently, their individual work rates add together directly:

$$\left(R_1 + R_2 + \ ...\ + R_n\right)T = 1$$

Substituting individual rate reciprocals $\left(\frac{1}{T_i}\right)$ into this model produces the classical work equation:

$$\left(\frac{1}{T_1} + \frac{1}{T_2} + \ ...\ + \frac{1}{T_n}\right)T = 1$$

**3. Sequential or Staggered Worker Inception**

In practical project management contexts, a primary worker may initiate a task alone for an isolated duration ($T_1$) before a secondary worker joins to accelerate completion over a shared remaining timeframe ($T$). This multi-stage operational lifecycle is modeled as:

$$R_1T_1 + \left(R_1 + R_2\right)T = 1$$

### Variation Relationships in Multi-Worker Environments

When a workforce expands or contracts, or when a project handles varying quantities of output units, we analyze the system using algebraic variation rules:

- **Inverse Relationship Between Labor Capacity (n) and Duration (T):** Assuming uniform individual work rates, increasing the number of active workers decreases the total time required to finish a fixed task. Thus, workforce size is inversely proportional to time $\left(n \,\alpha\, \frac{1}{T}\right)$.
- **Direct Relationship Between Job Volume (W) and Labor Capacity (n):** Scaling up the number of job units to be produced demands a proportionally larger number of workers for a static project time constraint $(W \,\alpha\, n)$.
- **Direct Relationship Between Job Volume (W) and Total Time (T):** Increasing the quantity of items to process extends the schedule length proportionally if the labor size is fixed $(W \,\alpha\, T)$.

### The Combined Work Formula

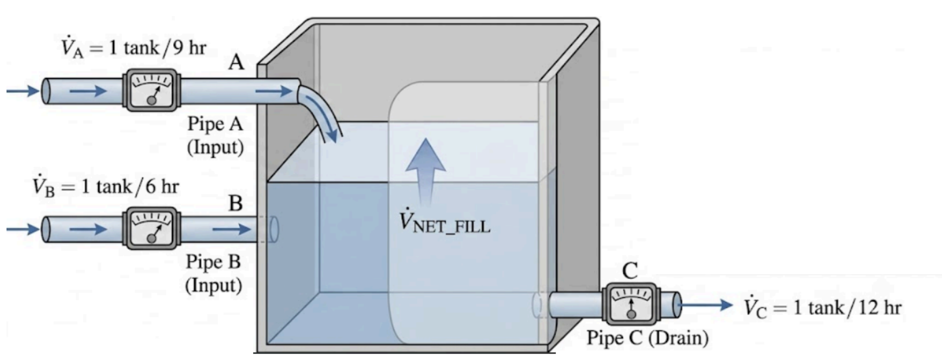
By merging these proportional dependencies into a singular **combined variation** expression, we derive the universal work invariant formula:

$$\frac{n_1 T_1}{W_1} = \frac{n_2 T_2}{W_2}$$

Where n represents workforce size, T is the working duration, and W is the number of distinct job units completed.

### Example

*"An input pipe A can fill a tank in 9 hours. Another input pipe B can fill the same empty tank in 6 hours. A drain pipe C can empty a full tank in 12 hours. If all the pipes are open, how long will it take them to fill the tank?"*



**Step 1: Define the Individual Performance Rates:**

- Inflow Rate of Pipe A: $R_A = \frac{1}{T_A} = \frac{1}{9}$ tanks per hour
- Inflow Rate of Pipe B: $R_B = \frac{1}{T_B} = \frac{1}{6}$ tanks per hour
- Outflow (Drain) Rate of Pipe C: $R_C = \frac{1}{T_C} = \frac{1}{12}$ tanks per hour

**Step 2: Establish Net System Directionality:** In fluid dynamics work problems, additions to the total asset volume are designated with positive coefficients, while subtractions from the asset volume (draining or leaks) are designated with negative coefficients.

$$\left(R_A + R_B + R_C\right)T = 1$$

**Step 3: Substitute the Parameter Rates:**

$$\left[\frac{1}{9} + \frac{1}{6} - \frac{1}{12}\right]T = 1$$

**Step 4: Isolate Total System Time (T):**

$$T = \frac{36}{7} \approx 5.1429\ hours$$

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# MOTION PROBLEM

**Motion problems** analyze the relationship between an object's position, its velocity, and time. These scenarios model real-world transit systems, logistics networks, and mechanical kinematics.

The universal governing equation for uniform linear motion is defined as:

$$Speed\ or\ Rate\ (v) = \frac{Distance\ (s)}{Time\ (t)}$$

$$s = vt$$

While calculating motion for a single leg of a journey is straightforward, complications arise when an object alters its speed across multiple intervals or executes a round trip. Managing these multi-stage journeys requires a firm grasp of **Average Speed (**$V_{ave}$**)**, a concept frequently tested—and misunderstood—on professional board exams.

**The Core Misconception of Average Speed**

The single most common mistake is assuming that average speed is simply the arithmetic mean of the individual speeds $\left(i.e., \frac{v_1 + v_2}{2}\right)$. This assumption is mathematically invalid because an object spends different amounts of time traveling at each speed if the intervals are defined by distance.

By definition, average speed is the total distance traveled divided by the total time elapsed:

$$V_{ave} = \frac{s_{Total}}{t_{Total}} = \frac{s_1 + s_2 + ... + s_n}{t_1 + t_2 + ... + t_n}$$

**Example:**

> ***"****Peter can walk from his house to his office at the rate of 5mph and back at the rate of 2mph. Find the average speed in mph."*

**Step 1: Define the Variables for Each Leg:** Let $s_{A\rightarrow B}$ be the distance from the house to the office, and $s_{B\rightarrow A}$ be the distance from the office back to the house. Since it is a round trip, these distances are perfectly equal ($s_{A\rightarrow B} = s_{B\rightarrow A} = s$).

- Outbound Speed ($v_1$): 5 mph
- Inbound Speed ($v_2$): 2 mph}

**Step 2: Express Time in Terms of Distance and Speed:** Since $t = \frac{s}{v}$, the time taken for each individual leg of the journey is:

- Outbound Time ($t_{A\rightarrow B}$): $\frac{s}{5}$
- Inbound Time ($t_{B\rightarrow A}$): $\frac{s}{2}$

**Step 3: Substitute into the Master Formula:** Construct the average speed fraction by summing the total distances and total times:

$$V_{ave} = \frac{s_{A\rightarrow B} + s_{B\rightarrow A}}{t_{A\rightarrow B} + t_{B\rightarrow A}} = \frac{2s}{\frac{s}{5} + \frac{s}{2}}$$

Notice that the actual distance $s$ completely cancels out! This proves that the average speed of a round trip is entirely independent of how far away the destination is.

**Step 4: Simplify the Complex Fraction:** Factor out the distance variable $s$ from the denominator to isolate the numerical rates:

$$V_{ave} = \frac{2}{\frac{7}{10}} \approx 2.8571\, mph$$

# INTEREST PROBLEM

**Interest problems** model financial transactions involving the time value of money. These scenarios analyze investments, business loans, and asset portfolios where capital yields an accumulated return over a designated timeframe.

The structural foundation of these problems relies on the basic definition of simple interest: the financial return generated is directly proportional to the initial capital invested, the length of the compounding period, and the periodic interest rate.

$$Interest\ (I) = Principal\ (P) \cdot Period\ or\ Time\ (t) \cdot Interest\ rate\ per\ period\ (r)$$

- **Principal (P):** The initial sum of money borrowed, loaned, or invested.
- **Time Period (t):** The duration for which the principal accumulates interest, typically expressed in years.
- **Interest Rate (r):** The fractional or percentage scaling factor applied to the principal per unit of time.

### Portfolio Diversification and Asset Allocation

The most common iteration of this topic on professional board exams involves a **mixture or allocation scenario**. Instead of analyzing a single static investment, a total sum of money is divided into two distinct sub-accounts, each yielding a different rate of return.

### The Modeling Strategy

When modeling a total capital fund ($P_{total}$) split across two financial instruments, we define the components using a single variable to avoid systems of equations:

- Let the allocation to the first account be: x
- The remaining balance allocated to the second account is automatically: $P_{total}$ - x

Because total interest income is additive, the combined income from both sub-accounts equals the target return:

$$Interest_1 + Interest_2 = Total\ Annual\ Income$$

$$\left(x \cdot r_1 \cdot t_1\right) + \left[\left(P_{total} - x\right) \cdot r_2 \cdot t_2\right] = Total\ Income$$

### Example:

> *"Ricardo loans an amount of ₱6,000, part at 80% annual interest and the rest at 20%. Calculate the amount of each loan if the total annual income is at ₱2,000."*

**<u>Step 1:</u> Define the Variable Spaces**

- Total Principal, $P_{total} = 6,000$
- Let $x = money\ loaned\ at\ 80\%\ interest$

- Let $6,000 - x = money\ loaned\ at\ 20\%\ interest$
- Time period, $t = 1$ year (since it specifies *annual* income)

**Step 2: Establish the Structural Equation**

$$Income\ from\ account\ 1 + Income\ from\ account\ 2 = Total\ Annual\ Income$$

$$(0.80)(x) + (0.20)(6,000 - x) = 2,000$$

**Step 3: Solve for x**

$$x = \frac{800}{0.60} = 1,333.33$$

**Step 4: Evaluate the Complementary Portfolio Balance**

- Principal allocated at 80%: $x = PhP1,333.33$
- Principal allocated at 20%: 6,000 - 1,333.33 = $PhP4,666.67$

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