

In engineering economic analysis, financial options are frequently presented with differing nominal interest rates and distinct compounding frequencies. To make a logically sound financial decision or comparison between multiple investment or credit instruments, an engineer must convert these varying structures into a common baseline. This baseline is established by finding **Equivalent Interest Rates**, which are defined as rates that yield the exact same accumulated future worth from an identical present principal over an identical total duration.

To equate two different nominal compounding structures, we leverage the concept of the **Effective Annual Interest Rate (i_e)**. The effective rate represents the true, real-world rate realized over a full 12-month calendar year. By setting the effective annual rate expression of the first financial option equal to the effective annual rate expression of the second option, we can solve directly for any unknown nominal parameters.

Find the nominal rate which if converted semi-annually could be used instead of 11% compounded bi-monthly.

To set up the governing equations, we break down the two compounding schemes presented in the text:

1. Given Configuration (The Baseline Rate)

- **Nominal Annual Interest Rate (r_1):** 11% = 0.11
- **Compounding Frequency (m_1):** "Bi-monthly" means compounding occurs once every two months. Because a standard year contains 12 months, the number of compounding sub-periods per year is:

$$m_1 = \frac{12 \text{ months}}{2 \text{ months}} = 6 \text{ periods per year}$$

2. Target Configuration (The Unknown Rate)

- **Nominal Annual Interest Rate (r_2):** The unknown value we need to find.
- **Compounding Frequency (m_2):** "Semi-annually" means compounding occurs twice a year:

$$m_2 = 2 \text{ periods per year}$$

The fundamental mathematical criterion for equivalence dictates that the effective annual rate of the semi-annual option must perfectly match the effective annual rate of the bi-monthly option ($i_{e,semi-annual} = i_{e,bi-monthly}$). The core effective rate formula is modeled as:

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Step 1: Set up the Equations

Equate the two effective rate models:

$$\left(1 + \frac{r_2}{m_2}\right)^m = \left(1 + \frac{r_1}{m_1}\right)^m$$

Substitute our known numerical values ($r_1 = 0.11$, $m_1 = 6$, and $m_2 = 2$) into the expression:

$$\left(1 + \frac{r_2}{2}\right)^2 = \left(1 + \frac{0.11}{6}\right)^6$$

Step 2: Simplify and solve simultaneously,

$$r_2 = 0.1121 \approx 11.21\%$$

In engineering economy, the relationship described above illustrates the core mathematical principle of **Financial Equivalence** under varying compounding frequencies. Formally, two different nominal interest rates paired with distinct compounding periods are considered economically equivalent if they yield the exact same **Effective Annual Interest Rate (i_e)**.

It means that if you deposit ₱10,000 today in a bank offering 11% compounded bi-monthly, and your friend deposits ₱10,000 in a bank offering 11.21% compounded semi-annually, both of you will walk away with the exact same amount of total cash at the end of the year. This mathematical relationship allows an engineer or investor to cut through marketing gimmicks and find the true, underlying productivity of capital across different financial systems.