Part A: ENGINEERING ECONOMY

The engineering economy deals with the concepts and techniques of analysis useful in evaluating the worth of systems, products, and services in relation to their cost. It serves as a bridge between physical engineering principles and economic feasibility, allowing practitioners to identify the alternative uses of limited resources and select the preferred course of action. While engineering evaluations rely heavily on mathematical models and cost data, professional judgment and experience remain vital inputs to account for real-world nuances.

To understand this discipline, one must distinguish between the physical engineering environment and the economic environment. The engineering environment is governed by physical laws—such as Boyle's law, Ohm's law, and Newton's laws of motion—and focuses on altering the physical environment to produce useful products and services. The objective here is physical efficiency, maximizing the output per unit of resource input:

$$Efficiency_{physical} = \frac{Output}{Input}$$

Conversely, the economic environment evaluates the worth of these products and services in terms of a medium of exchange, such as the peso or dollar. Economic efficiency measures the financial value generated relative to the cost incurred:

$$Efficiency_{economic} = \frac{Worth}{Cost}$$

Here, worth represents the annual revenue generated by operating a business, while cost represents the associated annual expenses.

Cost Classifications and Concepts

A thorough economic evaluation requires identifying and classifying different types of costs:

- **Fixed Costs:** Costs that remain unaffected by changes in the volume of output or activity over a specified range of operations. Examples include property taxes, insurance, executive salaries, and structural lease payments.
- **Variable Costs:** Costs that fluctuate in direct proportion to changes in production volume or activity level. Examples include raw materials, direct labor, and factory utilities tied directly to machine operations.
- **Incremental Cost (Marginal Cost):** The additional cost incurred from increasing production output or a specific activity by one unit. It is essential for determining the economic limit of scaling an operation.
- **Life-Cycle Cost:** The summation of all costs associated with a product, structure, or service throughout its entire life cycle. This spans research and development, design, production, operations, maintenance, and ultimate disposal or salvage.

- **Sunk Cost:** A cost that has already been incurred and cannot be recovered by any future decision or transaction. Sunk costs must be completely disregarded when choosing between future engineering alternatives, as they are unalterable historical facts. Common examples include past marketing studies, research and development expenditures, and non-refundable hiring bonuses.

The Concept of Money and Its Time Value

Money acts as a universally accepted medium of exchange for goods, services, and the repayment of debts. In financial environments, capital is not a free resource; a rental fee is charged by financial institutions for its use, which is designated as interest.

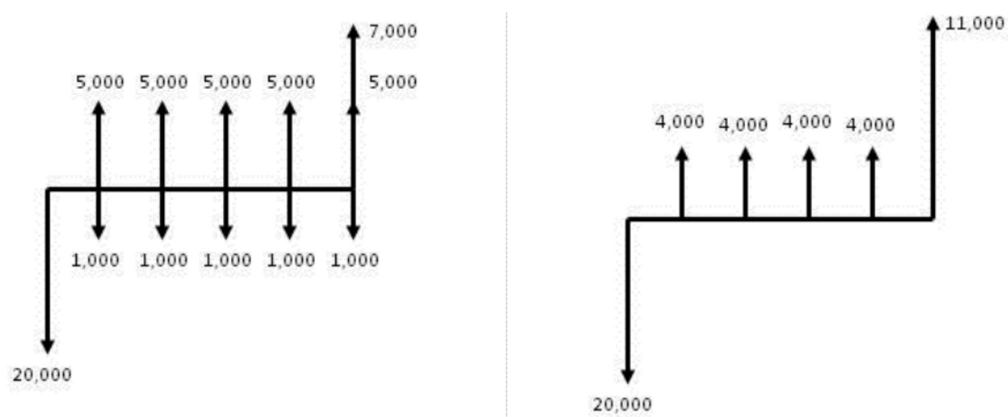
Because money has the capacity to earn interest over time through productive investment, a specific currency unit received at a future date does not possess the same economic value as that same unit held in hand today. This fundamental discrepancy establishes the **Time Value of Money**. Engineers must apply time-value principles to translate disparate past, present, and future cash flows into a single, common temporal baseline to conduct valid comparisons.

Cash Flow Diagrams

Graphical Representation of Cash Flows

A cash flow diagram is a graphical tool used to visualize and analyze financial transactions involving receipts (inflows) and disbursements (outflows) over time. These diagrams map complex cash schedules onto a structured timeline, facilitating a clear setup for engineering economy formulations.

For example, consider a mechanical device that will cost Php 20,000 when purchased. Maintenance will cost Php 1,000 each year. The device will generate revenues of Php 5,000 each year for five years, after which the salvage value is expected to be Php 7,000. The cash flow diagram is shown in the Figure



(a) Cash flow diagram (b) Simplified cash flow diagram

Cash flows can be evaluated from either the borrower's perspective or the lender's perspective; the diagram for one perspective is the exact inverse of the other.

Simple Interest

Principles of Simple Interest

Simple interest operates under the core principle that **only the original principal sum earns interest** over the lifetime of a transaction. Interest generated in previous periods is not added to the principal and does not earn additional interest in subsequent periods. This method is primarily applied to short-term loans or simple investment instruments where repayment occurs in a single lump sum.

The interest earned is directly proportional to the principal sum, the interest rate per period, and the total number of periods. The fundamental formula is:

$$I = P \cdot i \cdot n$$

Where: I = Total simple interest earned or paid
P = Present worth, principal, capital, or initial investment
i = Simple interest rate per annum (expressed as a decimal)
n = Total duration or number of interest periods (typically measured in years)

To determine the final accumulated total value, known as the Future Worth (F), the calculated simple interest is added directly back to the base principal

$$F = P + I$$

Substituting the interest equation into the accumulation formula yields:

$$F = P + (P \cdot i \cdot n)$$

$$F = P(1 + ni)$$

Types of Simple Interest

When time periods are stated in days rather than whole years, the value of n is formatted as a fraction: $\frac{Number\ of\ Days\ (d)}{Days\ in\ a\ Year}$. Depending on the regulatory or contractual framework, two distinct calculations are used:

1. **Ordinary Simple Interest:** Assumes a stylized commercial banker's year consisting of exactly **360 days**, divided into 12 uniform months of 30 days each. This convention yields a higher interest charge for a given number of days:

$$n = \frac{d}{360}$$

2. **Exact Simple Interest:** Utilizes the true astronomical calendar year consisting of exactly **365 days** (or 366 days during a leap year). This provides an accurate scientific accounting of time:

$$n = \frac{d}{365}$$

Compound Interest

Principles of Compound Interest

Unlike simple interest, compound interest dictates that **the interest earned in any period is added to the principal at the end of that period**. Consequently, in all subsequent periods, the newly accumulated interest earns interest alongside the original principal. This concept of "interest earning interest" forms the foundation of modern engineering economic analysis and long-term financial modeling.

Single-Payment Compound Interest Formulas

To track capital growth across n discrete compounding periods, compound interest formulas use exponential models rather than linear ones.

Future Worth Formula

To find the future value (F) of a present sum (P) after n periods at an interest rate of i per period:

$$F = P(1 + i)^n$$

The exponential term $(1 + i)^n$ is referred to as the *Single Payment Compound Amount Factor*.

Present Worth Formula

Conversely, to determine the value today (P) of a specific sum (F) that will be realized n periods in the future, solve for P:

$$P = F(1 + i)^{-n}$$

The term $(1 + i)^{-n}$ is known as the *Single Payment Present Worth Factor*.

Nominal versus Effective Interest Rates

Interest rates are often quoted on an annual basis, but compounding can occur multiple times within a single year (e.g., semi-annually, quarterly, monthly, or daily). This requires a distinction between nominal and effective interest rates.

Nominal Interest Rate (r)

The nominal rate is the basic annual interest rate quoted by financial institutions, which does not account for compounding within the year. It must always be adjusted to find the actual interest rate per compounding period (i):

$$i = \frac{r}{m}$$

Where:

- r = Nominal annual interest rate
- m = Number of compounding sub-periods per year (e.g., m=2 for semi-annual, m=4 for quarterly, m=12 for monthly)

Effective Interest Rate (i_e)

The effective rate represents the true annual rate realized over a full 12-month period, accounting for the compounding of interest within that year. It provides a standard metric for comparing different financial options with varying compounding frequencies. The formula is:

$$i_e = \left(1 + \frac{r}{m}\right)^m - 1$$

Discounts

In engineering economic studies and commercial transactions, **discount** represents the difference between the future worth (or face value) of a negotiable paper and its present worth. Put simply, it is the amount of interest that is deducted or collected in advance from a loan or financial obligation before the borrower receives the proceeds.

When a borrower secures a loan under a discount arrangement, the fund received (the present worth, P) is less than the total amount that must be repaid at maturity (the future worth, F). The financial difference represents the cost of borrowing, which is mathematically stated as:

$$D = F - P$$

Where: D = amount of discount
F = future worth, maturity value, or face value of the negotiable paper
P = present worth or principal proceeds received by the borrower

The Discount Rate (d)

The **discount rate** (d) is defined as the rate of discount of money per unit period of time. Visually and mathematically, it is expressed as the ratio of the total discount (D) deducted to the future worth (F) of the obligation:

$$d = \frac{D}{F}$$

By substituting the definition of discount ($D = F - P$) into this ratio, the relationship expands to:

$$d = \frac{F-P}{F}$$

This equation highlights that while a standard interest rate calculates the time value of money relative to the *present value*, a discount rate calculates the time value of money relative to the *future value*.

Deriving the Relationship Between Discount Rate (d) and Interest Rate (i)

To understand how a discount rate translates to an equivalent standard interest rate (i) for a single period, we examine their structural definitions. In standard simple interest, the future worth is related to the present worth by the formula F = P(1 + i), which conversely implies that the present worth can be discounted back using a negative exponent:

$$P = F(1 + i)^{-1}$$

Substituting this expression for P into the discount rate formula yields:

$$d = \frac{F-F(1+i)^{-1}}{F}$$

Simplifying yields:

$$d = \frac{i}{1+i}$$

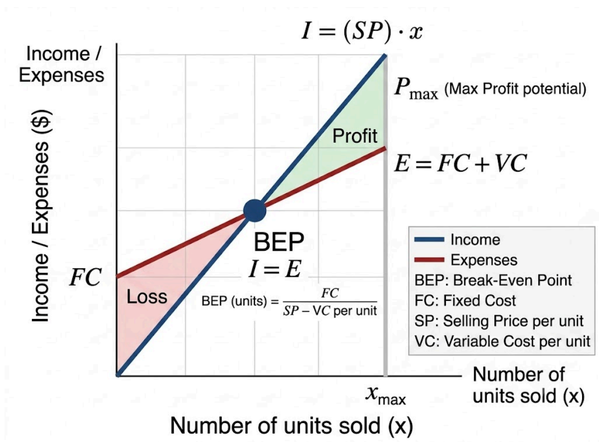
In **Engineering Economics and Banking**, a discount refers specifically to **interest collected in advance on a loan or negotiable paper**. This fundamental equation allows an engineer or financial analyst to convert a standard interest rate per period into an equivalent discount rate. Conversely, if the discount rate is known, the **equivalent interest rate** can be derived by rearranging the formula:

$$i = \frac{d}{1+d}$$

Under a discount setup, because the interest is paid upfront, the borrower effectively utilizes less capital than the nominal face value. Consequently, the effective interest rate (i) experienced by the borrower is always **higher** than the nominal discount rate (d).

Break-Even Analysis

Break-even analysis is a critical managerial and engineering economics tool used to evaluate the relationship between potential revenue, operational expenses, production volume, and the resulting profit or loss. It determines the specific operational threshold—known as the **Break-Even Point (BEP)**—where a business venture or manufacturing process shifts from generating a net loss to generating a net profit.



At the break-even point, the total income generated from operations exactly matches the total expenses incurred, resulting in a net profit of zero:

$$Income\ (I) = Expenses\ (E)$$

Income (I)

Income represents the total revenue generated from selling a specific quantity of goods or services. It is a linear function directly proportional to the volume of units sold, mathematically expressed as:

$$I = SP(x)$$

Where:

- I = total income or revenue
- SP = selling price per individual unit
- x = number of units produced and sold

Expenses (E)

Total operational expenses are composed of two distinct cost behaviors: fixed costs and variable costs.

$$E = FC + VC$$

Where:

- FC = **Fixed Costs**. These are expenditures incurred by a firm that remain constant regardless of production output. Even if production drops to zero, fixed costs must still be satisfied. Examples include structural depreciation, property insurance, administrative salaries, and facility rent.
- VC = **Variable Costs**. These costs fluctuate in direct proportion to the volume of production. Variable costs scale linearly with the number of units manufactured and typically encompass raw material costs, direct factory labor, and manufacturing utilities. Variable costs can be broken down as $VC = (vc)(x)$, where vc represents the variable cost per unit.

Therefore, the complete equation for total expenses is:

$$E = FC + (vc)(x)$$

Mathematical Derivation of the Break-Even Quantity (x_{BEP})

To find the exact volume of units (x) required to break even, equate the total income formula to the total expenses formula:

$$I = E$$

$$SP(x) = FC + (vc)(x)$$

Solving simultaneously;

$$x_{BEP} = \frac{FC}{SP - vc}$$

The denominator, $(SP - vc)$, is professionally referred to as the **unit contribution margin**. It represents the amount that each unit sold contributes toward covering the fixed operational expenses of the firm. Once the accumulated contribution margins match the fixed costs, the break-even point is achieved, and any unit sold beyond x_{BEP} yields a direct profit.