

To solve for the volumetric efficiency of a positive-displacement reciprocating pump, we evaluate the geometric boundary conditions and physical fluid movement inside the piston cylinder assembly. Unlike dynamic centrifugal pumps that rely on fluid velocity scaling, a reciprocating pump operates by trapping a fixed volume of liquid within its cylinder cavity and forcing it out mechanically via a driven piston or plunger.

The problem provides a single-acting reciprocating pump with a cylinder bore diameter ( $d$ ) of 360 mm (0.360 m) and a piston stroke length ( $L$ ) of 400 mm (0.400 m), operating at a rotational speed ( $n$ ) of 380 rpm. It states that the fluid leakage profile yields a percent slip (%Slip) of 12%.

To understand the core relationship, we look at the definitions of **Ideal Discharge ( $V_D$ )** and **Actual Discharge ( $Q$ )**. The ideal or displacement volume flow rate represents the total volume swept by the piston per unit time under zero-leakage conditions. For a single-acting pump where the piston rod area is neglected, it is expressed as:

$$V_D = \frac{\pi}{4} d^2 \cdot L \cdot \frac{n}{60}$$

### **Step 1: Compute the Ideal Displacement Discharge Rate ( $V_D$ )**

Substitute the geometric parameters into the displacement equation, ensuring all dimensions are uniformly converted to standard meters and seconds:

$$V_D = \frac{\pi}{4} (0.360m)^2 \cdot (0.400m) \cdot \frac{380 \text{ rpm}}{60}$$

$$V_D = 0.6447 \frac{m^3}{s}$$

### **Step 2: Determine Volumetric Efficiency ( $\eta_v$ ) via the Slippage Concept**

By fundamental definition, **Slip** is the difference between the ideal theoretical displacement and the actual fluid volume delivered through the discharge check valves ( $\text{Slip} = V_D - Q$ ). Expressed as a ratio relative to total displacement, the percentage of slip is given by:

$$\%Slip = \frac{V_D - Q}{V_D} = 1 - \frac{Q}{V_D}$$

Volumetric Efficiency ( $\eta_v$ ) is defined as the ratio of actual volume discharged to the ideal theoretical displacement volume:

$$\eta_v = \frac{Q}{V_D}$$

Substituting the volumetric efficiency term directly into the percentage slip equation reveals a foundational relationship:

$$\%Slip = 1 - \eta_v$$

$$\eta_v = 1 - 0.12 = 88\%$$

In positive-displacement machinery, a slip value of 12% indicates that 12% of the theoretical fluid volume is escaping back through the internal clearances. This fluid bypass typically occurs during the brief moments when the suction and discharge check valves are transitioning open or closed, or via high-pressure leakage past the piston rings and packing glands.

If a pump in an operating plant shows a sudden increase in slip (and a corresponding drop in volumetric efficiency), it serves as a diagnostic warning that the check valve seats are eroding or the piston seals are wearing out and require immediate maintenance.

This problem highlights an important strategy for board exams: distinguishing between necessary parameters and intentional distractors. The author included the bore, stroke, and speed to tempt students into performing a long calculation sequence.